

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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## **Mini-Workshop: Boundary Value Problems and Spectral Geometry**

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**ABSTRACT.** Boundary value problems and spectral geometry is an attractive and rapidly developing area in modern mathematical analysis. The interaction of PDE methods with concepts from operator theory and differential geometry is particularly challenging and leads directly to new insights and applications in various branches of pure and applied mathematics, e.g., analysis on manifolds, global analysis and mathematical physics. Some recent contributions in the field of boundary value problems and spectral geometry concern, e.g., construction of isospectral manifolds with boundary, eigenvalue and resonance distribution for large energies, multidimensional inverse spectral problems, singular perturbations, new regularity techniques, Dirichlet-to-Neumann maps and Titchmarsh-Weyl functions.

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### **Introduction by the Organisers**

The basic aim of the mini-workshop is to bring together a special selection of world leading experts from different areas, as, e.g., spectral theory, differential geometry, analysis of PDEs, mathematical imaging, to join the efforts in studying several problems arising in the modern mathematical physics.

In various situations one deals with the spectral or scattering analysis of complex objects which are built together from elementary pieces interacting with each other through the boundary. The elementary pieces may have rather simple properties and admit an explicit description, and the properties of the total system come mostly from the global geometry or the interaction conditions. The situation can be modeled in many ways, for example, there is a considerable progress

in understanding the properties of systems composed from one-dimensional pieces (differential operators on metric graphs), both the direct and the inverse spectral theory are in active development. A more difficult problem is to study differential operators on coupled domains or manifolds. It is known that domains of some special geometries can be approximated by metric graphs and hence one is interested in the question if the same research philosophy can be transferred from metric graphs to more complicated coupled objects. Even a partial progress in this direction needs combining various techniques like spectral analysis of operator pencils, trace and embedding theorems, pseudodifferential operators, differential geometry and many others. During the last years there were several attempts to fill the gap, in particular, by developing operator-theoretical tools suitable for studying rather general boundary value problems for PDEs.

Nowadays many of the new trends in the field of boundary value problems and spectral geometry develop rapidly and independently into different directions of modern analysis. A special feature of the mini-workshop is to combine the expertise of colleagues from these different areas and to focus on the following topics:

- Boundary value problems with low regularity (rough domains, singular spaces, mixed boundary value problems);
- Decomposition techniques for composed domains and understanding the relation between local and global properties;
- Operator-theoretical tools like abstract boundary triples in spectral geometry,
- Spectral analysis via Dirichlet-to-Neumann maps and Titchmarsh-Weyl functions.

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