

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mini-Workshop: Manifolds with Lower Curvature Bounds

Organised by
Anand Dessai, Fribourg
Wilderich Tuschmann, Karlsruhe
Burkhard Wilking, Münster

January 1st – January 7th, 2012

ABSTRACT. The purpose of the meeting was to relate and study new developments in the geometry and topology of Riemannian manifolds with lower curvature bounds. Special emphasis was given to lower Ricci curvature bounds in the sense of Lott-Villani and Sturm and to the gradient flow on metric spaces as well as to manifolds with a lower sectional curvature bound.

Mathematics Subject Classification (2000): 53: Differential Geometry, 53C20: Global Riemannian geometry, including pinching, 53C21: Methods of Riemannian geometry, including PDE methods; curvature restrictions.

Introduction by the Organisers

The workshop *Manifolds with Lower Curvature Bounds*, organised by Anand Dessai (Fribourg), Wilderich Tuschmann (Karlsruhe) and Burkhard Wilking (Münster), was held January 1st–January 7th, 2012. The meeting was attended by 17 participants, ranging from second year graduate students to senior researchers.

The purpose of the meeting was to relate and study new developments in the geometry and topology of Riemannian manifolds with lower curvature bounds. Special emphasis was given to manifolds with given lower bounds on Ricci curvature as well as to manifolds of nonnegative/positive sectional curvature.

The meeting was organised around ten one-hour talks, two mini courses, and two short talks by young PhD students, leaving plenty of time between and after talks for informal discussions.

The workshop started off with a talk by Guofang Wei surveying various characterizations of lower Ricci curvature bounds. Later on in the week Nicola Gigli (Oberwolfach Prize Winner 2010) and Giuseppe Savaré each gave 2 hour mini

courses on optimal transport, lower Ricci curvature bounds and their work on gradient flows on metric spaces thereby also providing a gentle introduction to the recent work of Lott-Villani and Sturm. David Wraith described new methods to construct invariant metrics of positive Ricci curvature on G -manifolds with finitely many non-principal orbits.

Other talks were related to lower bounds on sectional curvature (curvature for short). In his talk Igor Belegradek considered the space $\mathcal{R}(N)$ of complete metrics of nonnegative curvature on an open connected manifold N and showed in particular that the complement of any countable subset of $\mathcal{R}(\mathbb{R}^2)$ is path connected. Dmitri Panov discussed a polyhedral analogue of Frankel's conjecture and supporting evidence for it in low dimensions. Comparison theorems for integrals and Hölder norms under curvature bounds in the absence of injectivity radius bounds were addressed in Patrick Ghanaat's talk. Marco Radeschi showed that singular Riemannian foliations of the round sphere with leaves of dimension ≤ 3 are homogeneous. Computations for the algebra of stable polynomial invariants of Riemannian manifolds and speculations on how to describe it by graphs using a Rozansky-Witten approach were discussed by Gregor Weingart.

Another bulk of talks was devoted to the topology of Riemannian manifolds with nonnegative/positive sectional curvature and large symmetry. Fernando Galaz-Garcia and Wolfgang Spindeler discussed the (equivariant) classification of nonnegatively curved manifolds of dimension 4 and 5 with isometric action of the 2-torus and nonnegatively curved fixed point homogeneous 5-manifolds. Lee Kennard explained his use of the Steenrod algebra to prove generalized Hopf conjectures for positively curved manifolds under logarithmic lower bounds on the symmetry rank.

The meeting also included two short talks by young PhD students. Martin Herrmann presented a criterion for the total space of principal bundles to admit almost nonnegative curvature operators. Nicolas Weisskopf discussed a conjecture on the strong rigidity of the elliptic genus for positively curved spin manifolds with symmetry.

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