Abstract. The meeting focused on the last advances in the applications of variational methods to evolution problems governed by partial differential equations. The talks covered a broad range of topics, including large deviation and variational principles, rate-independent evolutions and gradient flows, heat flows in metric-measure spaces, propagation of fracture, applications of optimal transport and entropy-entropy dissipation methods, phase-transitions, viscous approximation, and singular-perturbation problems.

Mathematics Subject Classification (2000): 30Lxx, 35A15, 35Qxx, 49xx, 74xx, 76xx.

Introduction by the Organisers

It is well known that the study of many important evolution problems gains benefit by adopting a variational point of view. Variational methods can be quite helpful to better understand the intimate structure of the problem, to derive new properties concerning existence, uniqueness, stability, approximation, and long-time behaviour, to guess new estimates, to find the right or more appropriate formulations, to produce new kind of solutions, to explain the relationships between different approaches or techniques, to derive or justify some equations in terms of general principles and stochastic models, to study the stability of a system in terms of a set of parameters or to capture its behavior in a singular perturbation limit.

The workshop, organized by Alexander Mielke (WIAS, Berlin), Felix Otto (Max Planck Institute, Leipzig), Giuseppe Savaré (Univ. Pavia) and Ulisse Stefanelli
(IMATI-CNR, Pavia), aimed to present many new, striking and promising achievements in this wide area, thanks to the contribution of over 50 participants with broad geographic representation and a variety of research fields, each revealing different methodology, interests, and level of abstraction.

One of the organizers’ main goal was to generate a strong interaction between various subjects and people with different backgrounds, covering calculus of variations, optimal transport, phase transitions, fluid mechanics, materials science, stochastic calculus, and models. Approximately 25 talks of varying lengths were delivered by experts in the fields but also by quite a number of young post-docs and PhD students and stimulated a lot of discussions in a friendly and inspiring atmosphere, that contributed to the overall success of the meeting.

Among the main themes presented during the workshop, we quote here

- the interplay between the theory of large deviation in stochastic analysis on the one hand and variational principles on the other; e. g. an interpretation of the Wasserstein gradient flow structure in terms of large deviation, rigorous low-temperature bounds for the relaxation in multi-well energy landscapes, singular limits of rate functionals;
- the energetic approach to rate-independent evolution problems and various viscous regularizations, with applications to damage, fracture, and phase transitions;
- crack propagation, e. g. an emerging analysis of fully dynamic models, novel functional analytic aspects of the quasi-stationary models, and the analysis of alternative propagation criteria;
- the classic or new gradient-flow structure of various linear and nonlinear PDE’s;
- the application of optimal transport techniques to solve, study, or approximate efficiently evolution equations from fluid mechanics, particle interaction, quantum problems;
- entropy-entropy dissipation methods for evolution and their link with sharp functional inequalities;
- the interplay between evolution problems and geometry in metric-measure spaces; e. g. a clear distinction between Riemannian and Finsler settings is emerging;
- phase-transitions, viscous approximation, and singular-perturbation problems.

Special event

On Thursday afternoon, the Oberwolfach Prize was awarded to Nicola Gigli and László Székelyhidi for their excellent achievements in Analysis and Applied Mathematics. The award presentation by Gert-Martin Greuel, Director of the Institute, and by Manfred Feilmeier of the Oberwolfach Foundation, was followed by the Laudationes of the two price-winners. Luigi Ambrosio explained the crucial contributions of Gigli concerning the metric theory of gradient flows, the structure of the Wasserstein space, and the heat flow in non-smooth metric-measure spaces.
Stefan Müller explained the fundamental new insights of Székelyhidi into the theory of nonlinear partial differential equations and their applications in continuum mechanics, focusing in particular on compensated compactness, partial regularity and its failure.

Nicola Gigli then gave a lecture on his more recent results on metric measure spaces with a lower Ricci curvature bound and László Székelyhidi presented his recent achievements on the Euler equations, in particular the Onsager conjecture, and its relation to the Nash-Kuiper embedding in geometry. In the evening, there was a festive dinner.
Workshop: Variational Methods for Evolution

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