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Correlations and Interactions for Random Quantum Systems

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Abstract. Random quantum systems cover a broad range of mathematical models from random Schrödinger operators to random matrices and quantum spin models with random parameters. Their understanding requires techniques which combine functional analysis and probability. The workshop brought together researchers from these various branches which discussed new results, methods and future challenges. This is a report on the meeting containing extended abstracts of the lectures.

Mathematics Subject Classification (2000): 47N50, 60K40, 81V70.

Introduction by the Organisers

The half-size workshop on *Correlations and Interactions for Random Quantum Systems* was organised by Peter D. Hislop (Lexington, KY), Werner Kirsch (Hagen), Peter Müller (München) and Simone Warzel (München). It was attended by 30 participants from Canada, Japan, the US and various European countries. The program consisted of 22 lectures covering new results, recent developments and future challenges in the field. Special attention was paid throughout to providing a platform for younger researchers. This report contains extended abstracts of these lectures. On behalf of all participants, the organisers would like to thank the staff and the director of the Mathematisches Forschungsinstitut Oberwolfach for providing such a stimulating and inspiring atmosphere.

Mathematical research on random quantum systems covers various branches: the theory of random Schrödinger operators, random matrices and the analysis of

models in quantum statistical mechanics with random parameters. The common theme is to describe localization phenomena arising in different branches of quantum physics from condensed matter theory to quantum information processing. While being motivated by physics, these models pose interesting mathematical challenges in themselves which call for a combination of ideas from functional analysis to probability. The interplay of mathematical disciplines in this area is nicely illustrated by the topics covered in this workshop. For examples, A. Klein presented a joint work with J. Bourgain on a 30-year-old problem concerning the regularity of the density of states of rather general Schrödinger operators which need not be random. On the other hand, B. Virag gave a lecture on his recent results pertaining to the Brownian corousel, a stochastic processes which is related to the random process of eigenvalues of the Anderson model in dimension $d = 1$ in a weak disorder limit.

The Anderson model is the prototypical example of a random Schrödinger operator describing a single quantum particle in a random potential. One of its striking features is the occurrence of a dense point spectrum and the absence of transport for $d = 1$ and in higher dimension close to band edges or for strong disorder. For low disorder and $d > 2$, physicists expect a region with absolutely continuous spectrum and diffusive transport. This remains an open challenge for mathematicians. Recent progress related to proofs of transport in albeit different models than the Anderson model were presented at the meeting (e.g. in the talks by A. Joye and J. Schenker).

One other challenge in the field is to step beyond the framework of single-particle operators and investigate the localization properties of systems of many interacting particles. Some progress has been made in this direction: for a fixed, but finite number of particles the localization regime in the Anderson model was proven to be stable under finite-range interactions. However, for systems of infinitely many particles with a positive density, the precise formulation and stability of the localization regime generally remains an open problem. All the more remarkable are therefore recent results for special systems such as the one related to the Quantum Hall Effect and certain integrable models of quantum statistical mechanics in $d = 1$. The latter were presented at the workshop in the talks of B. Sims and G. Stolz. Moreover, these talks were conveniently framed by survey talks of B. Nachtergaele and L. Pastur explaining the greater challenges and questions to the community emerging from the field of quantum information theory and quantum statistical mechanics.

Workshop: Correlations and Interactions for Random Quantum Systems

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