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Applied Dynamics and Geometric Mechanics

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Abstract. This one week workshop was organized around several central subjects in applied dynamics and geometric mechanics. The specific organization with afternoons free for discussion led to intense exchanges of ideas. Bridges were forged between researchers representing different fields. Links were established between pure mathematical ideas and applications. The meeting was not restricted to any particular application area. One of the main goals of the meeting, like most others in this series for the past twenty years, has been to facilitate cross fertilization between various areas of mathematics, physics, and engineering. New collaborative projects emerged due to this meeting.

The workshop was well attended with participants from Europe, North America, and Asia. Young researchers (doctoral students, postdocs, junior faculty) formed about 30% of the participants.

Mathematics Subject Classification (2000): 37.-xx, 58.-xx, 70.-xx, 74.-xx,76.-xx, 85.-xx.

Introduction by the Organisers

This workshop on applied dynamics and geometric mechanics was introduced over 20 years ago by the late Klaus Kirchgässner and the late Jerrold E. Marsden. After Kirchgässner's retirement, this workshop continued to take place at regular intervals of three years under the leadership of Jerrold E. Marsden and Jürgen Scheurle. Every such meeting was inspired by new developments in the core fields of dynamical systems and geometric mechanics as well as recent exciting applications. The present meeting continued this tradition. The whole subject is scientifically extremely active and progressing in various new and promising directions. Each speaker and invitee presented a new mathematical or computational tool in dynamical systems or geometric mechanics and linked it either to applications, or forged a bridge between various areas of pure dynamical systems and geometric mechanics and certain applications to physics, engineering, and other basic sciences.

A broad overview of these topics as covered in the meeting is as follows:

Dynamical Systems. The basic theory of dynamical systems is developing with numerous new ideas that have importance in applications. The meeting presented recent advances in this theory with an emphasis on applications in one of the following specific areas.

• *Astrodynamics.* There is a rich history of applications of the theory of dynamical systems to problems in astrophysics. The morning of the first day of the workshop was devoted entirely to problems in astrophysics and celestial mechanics. The speakers and posters emphasized the link between their own research problems to branches of dynamical systems and geometric mechanics.

• *Control Theory.* An important area of research that is closely connected with both dynamical systems and geometric mechanics is nonlinear control theory. Nonlinear control is naturally formulated on manifolds and has particular applications to mechanical systems where the control is studied on a Riemannian manifold. Basic problems include controllability, stabilization, and optimal control. A recent area of great interest is the simultaneous control of multiple systems (swarms). Such systems include groups of wheeled or legged robots or groups of submarine robotic vehicles. There are very interesting connections of this work with the study of biological swarming in birds or fish, for example. Another important topic is the control of interconnected systems which can be studied using the theory of Dirac structures. This is important for both electrical and mechanical networks. Several talks and posters on these topics were given during the workshop.

• *Multi-Agent Systems.* Many modern technical systems, such as multiple mobile robots in the same workspace, are composed out of many, relatively simple, subsystems (agents) that interact in a complicated way. Since the computational limitations of dealing with such complex interconnected systems have already been reached, new methods are being developed that treat such systems as strongly and weakly interconnected units. In other words, instead of thinking of these systems as huge systems of differential equations (ordinary or partial), one is organizing them in smaller units with massive message passing and information exchange between them. The speakers and posters presented new tools that could lead to the automatic determination of the strong or weak coupling between such subsystems; this then naturally leads to a hierarchy of corresponding dynamical computations. Another topic presented was about the development of cooperative distributed control strategies, both at the theoretical and computational/numerical level.

Geometric Mechanics. By now, geometric mechanics has matured to a main subject at the interface between several areas such as symplectic, Poisson, and Dirac geometry, dynamical systems, variational calculus, theoretical physics, numerical analysis, control theory, and various areas of engineering science. The subject continues to thrive and develop, both by widening its core to include other theoretical fields such as integrable systems and conservative evolutionary PDEs and by reaching out to new applied areas such as imaging science, discrete differential geometry, stochastic analysis, or materials science. The following core subjects were specifically addressed at this workshop.

• *Nonholonomic Mechanics.* Nonholonomic mechanics is an important extension of Hamiltonian mechanics whose goal is the study of the dynamics of mechanical systems subject to nonintegrable constraints on the velocity (such as the rolling wheel, the ice skate, or the rolling ball). Such systems are endowed with a bracket that does not satisfy the Jacobi identity and have a dynamics that non-trivially generalizes that seen in Hamiltonian systems. For example, volume is not necessarily conserved in the phase space and it is possible to get asymptotic stability even in the absence of external friction. Also, there is an interesting generalization of Noether's theorem: Symmetry does not usually lead to momentum conservation but to a dynamic momentum equation. Some systems admit a "Hamiltonization", that is, one can produce a Poisson bracket such that relative to it and after a time reparametrization, the system becomes Hamiltonian. This then allows the study of the integrability of nonholonomic systems. There is also an interesting connection with control theory because nonintegrable constraints are related to the fact that a system remains controllable in the presence of constraints. There are many applications to robotic problems and problems in submarine and flight dynamics using these ideas. Several talks and posters addressed these fundamental questions and linked them to many applications.

• *Discrete Mechanics.* Discrete mechanics and the numerical analysis related to this area was another core subject represented at this workshop. Since the late 1980's, the field of geometric integration and structure preserving algorithms has seen spectacular development. It has produced and analyzed numerical methods for ordinary differential equations and, more recently, for partial differential equations, that preserve exactly (i.e., up to round-off error) as much of the underlying geometric structure as possible. Geometric mechanics has been one of the main beneficiaries of these geometric integration techniques which has led to new developments in the simulation of mechanical systems in particle and continuum mechanics as well as their stochastic counterparts. Several talks and posters addressed the above mentioned problems.

• *Stochastic Mechanics.* Randomness is ubiquitous in the description of dynamical phenomena for several reasons. It may express our lack of knowledge about the systems' parameters or components. It may arise intrinsically as non-determinism in a large spectrum of areas ranging from statistical mechanics to interacting systems of agents on financial markets. It also appears as an analytical tool in area of mathematics such as control theory and differential geometry. It may also be encountered as noise in mesoscopic limits of dynamical systems on different scales, arising in many different ways, from turbulence in ocean-atmosphere dynamics, to order book fluctuations governing price dynamics, in which the small scale component has good mixing properties. The talks and posters representing this area underlined the achievements recently obtained in financial mathematics as well as stochastically perturbed conservative systems. In addition, the geometry induced by the addition of noise to a deterministic dynamical system was also presented.

• *Liquid Crystals.* The mathematical study of complex materials has seen an explosive growth in the past few years. These materials, both fluids and solids, are characterized by an additional internal structure of the particles which is encoded in an order parameter group. Liquid crystals are important representatives of complex materials. The dynamics of liquid crystals is dominated by two theories: Ericksen-Leslie order parameter theory and Eringen micropolar theory. No link between these theories has been established, in spite of the fact that the latter should naturally include the former. Worse, all attempts at proving this implication have failed due to errors of all papers addressing this problem in the literature. At this workshop, using sophisticated tools of geometric mechanics, a theorem was presented that proves such an inclusion.

Structure of the Meeting. Consistent with the general policy of Oberwolfach there were only twenty main lectures at the meeting. Twenty invitees were asked to speak and present their latest results. All participants were invited to present a poster. There were slots in the schedule specifically for advertising and viewing the posters, respectively, (teaser and presentation sessions). Wednesday evening an open discussion was held addressing the recent developments in the field of Geometric Mechanics and the new directions where it is supposed to develop.

Poster titles and poster presenters. The following posters have been presented at the workshop.

• Twisted structures in nonholonomic systems (Paula Balseiro)
• Stability of stationary fronts in inhomogeneous wave equations (Gianne Derks)
• Continuous and discrete Neumann systems on Stiefel varieties (Yuri Fedorov)
• Applications of Hamiltonization of nonholonomic systems (Oscar Fernandez)
• Solitary waves in a chain of coupled Fitzhugh-Nagumo neurons (Andreas Johann)
• Optimizing the stable behaviour of controlled dynamical systems (Peter Koltai)
• Global symplectic uncertainty propagation on Lie groups (Melvin Leok)
• Discrete Dirac mechanics and discrete Dirac geometry (Melvin Leok, Tomoki Ohsawa)
• Routh reduction for singular Lagrangians (Bavo Langerock)
• Moving framework and fiber bundle methods in nonholonomic mechanics (Jared Michael Maruskin)
• Involutive distributions and dynamical systems of second order type (Tom Mestag)
• Variational integration of constrained dynamics on different time scales (Sina Ober-Blöbaum, Sigrid Leyendecker)
• Nonholonomic Hamilton-Jacobi theory (Tomoki Ohsawa)
• On the topology of the double spherical pendulum (Manuele Santoprete)
• Discrete integrable dynamical systems (Yuri B. Suris)
• Invariant sets forced by symmetry (Sebastian Walcher)

Geometric Mechanics quo vadis? The discussion on Wednesday evening was one of the most interesting aspects of the meeting. Many topics that have become central to geometric mechanics or that use its tools in an essential way were identified. Here are some that were mentioned in this open discussion: Interplanetary missions, variational integrators, swimming theory, Lagrangian coherent structures, Euler-Poincaré theory, Lie-Poisson reduction, multisymplectic integrators, nonlinear stability, underwater vehicles, geometrical optimal control, computational anatomy, reduction by stages in both the Hamiltonian and Lagrangian setting, molecular oscillations, dynamics of asteroids pairs, dynamics of satellite with tethers, molecular strand theory, geometrically exact elasticity, robotics, solitons and peakons, various aspects of fluid dynamics, turbulence models, geometric formulation of complex fluid theory, liquid crystals, superfluids, plasmas, magnetohydrodynamics, geophysical fluid dynamics, general relativity, field theory, Lie groupoids and algebroids, swarming theory, telecommunications.

Several main directions for geometric mechanics were identified in this discussion:

• How does one deal with low regularity in geometric mechanics? The obvious example is compressible barotropic fluid flow that can be formally written as a Hamiltonian system, yet, after the first shock, it can be rigorously proven that the energy decays.

• Symmetric Hamiltonian Bifurcation Theory. Very little is known about this subject which remains, to these days, almost totally underdeveloped. The challenge here is to bring it to the level of the by now standard theory of symmetric bifurcation for generic vector fields developed by Golubitsky and his collaborators. One of the main technical tools in this development is the geometry of the momentum map.

• The Nature of Integrability. In spite of spectacular developments in the theory of integrable systems, the field itself is poorly developed. This entire area rests essentially on large classes of known integrable ODEs and PDEs. While Fomenko and his collaborators have developed a topological classification method, almost nothing is known about a symplectic classification. This in turn is essential in the study of semiclassical quantization and spectral analysis.

• Hybrid discrete and continuous systems. Hybrid systems can be described mathematically by a mixture of logic based switching and difference/ differential equations. Also, stochastic components can be included. Besides of continuously varying variables and parameters such systems additionally contain variables with a discrete range of values. Variations of these lead to sudden structural changes of the systems and thus to a rapid change in the systems' behaviour. Many systems in engineering and some physical systems can be modeled using such a mathematical framework. Examples are contact problems in mechanics, event driven systems and adaptive control. A general theory for this remains to be developed.

• Networks with varying connectivity/ topology. Complex networks play a central role in today's society. For example, communication, mobility and transportation are based and rely on networks. The question of how the network topology affects the performance has not been addressed systematically. The topology of a network is determined by the nature of the coupling between its nodes (connectivity). The character of the coupling can change when structural parameters are varied. An interesting research topic is the extension of classical concepts from bifurcation and stability theory to such systems.

Workshop: Applied Dynamics and Geometric Mechanics

Table of Contents

Fred C. Adams (joint with Anthony M. Bloch)
Astrophysical Orbits and Random Hill's Equations 2225

John Baillieul (joint with Wing-Shing Wong)
Brockett-Heisenberg Systems and Geometric Aspects of Control Communication Complexity 2227

James Binney
Torus Dynamics 2228

David Martín de Diego
New Developments in Discrete Variational Calculus and Discrete Optimal Control Theory 2230

David Elworthy (joint with Yves LeJan and Xue-Mei Li)
Geometry of Adding Noise to Dynamical Systems, and Decomposition of the Result 2232

Luis García Naranjo (joint with Paula Balseiro)
Gauge Transformations, Twisted Poisson Brackets and Hamiltonization of Nonholonomic Systems 2235

François Gay-Balmaz (joint with T. S. Ratiu and C. Tronci)
Equivalent Theories of Liquid Crystal Dynamics 2238

Darryl D. Holm (joint with Colin J. Cotter, Rossen I. Ivanov and James R. Percival)
Waltzing Peakons and G-Strands 2241

Arieh Iserles (joint with María José Cantero)
Rapid Expansions in Orthogonal Polynomials 2242

Oliver Junge (joint with Gary Froyland and Péter Koltai)
Estimating Long Term Behavior of Flows Without Trajectory Integration: The Infinitesimal Generator Approach 2244

P. S. Krishnaprasad
Control of Collectives 2247

Melvin Leok (joint with Tatiana Shingel)
General Techniques for Constructing Variational Integrators 2250

Sigrid Leyendecker and Sina Ober-Blöbaum
A Variational Approach to Multirate Integration for Constrained Systems 2253

Juan C. Marrero (joint with Yuri N. Fedorov and Luis García-Naranjo)
Invariant Measures in Nonholonomic Mechanics 2256

Alexander Mielke
Geometry and Thermodynamics for the Coupling of Quantum Mechanics and Dissipative Systems 2260

James Montaldi (joint with Mark Roberts and Frédéric Laurent-Polz)
A Zoo of Bifurcations in Point Vortex Systems on the Sphere 2263

Sebastian Reich (joint with Georg Gottwald, Kay Bergemann, Eugenia Kalnay, Javier Amezcua and Kayo Ide)
Dynamical Systems Approach to Data Assimilation for Mechanical Systems 2266

Daniel J. Scheeres
Minimum Energy Configurations in the N-Body Problem and the Celestial Mechanics of Granular Systems 2269

Josef Teichmann
Geometry of Option Price Surface Models 2271

Hiroaki Yoshimura (joint with Henry Jacobs)
Interconnection, Dirac Structures and Dirac Systems in Mechanics 2273

Dmitry Zenkov (joint with Cameron Lynch and Kenneth Ball)
Stability of Relative Equilibria of Discrete Chaplygin Systems 2275