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Real Analysis, Harmonic Analysis and Applications

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Abstract. The workshop has focused on important developments within the last few years in the point of view and methods of real and harmonic Analysis as well as significant concurrent progress in the application of these to various other fields.

Mathematics Subject Classification (2000): 42xx, 43xx, 44xx, 22xx, 35xx.

Introduction by the Organisers

This workshop, which continued the triennial series at Oberwolfach on Real and Harmonic Analysis that started in 1986, has brought together experts and young scientists working in harmonic analysis and its applications (such as arithmetic combinatorics, quasiconformal mappings, nonlinear dispersive and elliptic PDE, and ergodic theory) with the objective of furthering the important interactions between these fields.

Major areas and results represented at the workshop are:

- The use of the polynomial method in harmonic analysis has recently led to advancement on several classical problems such as the multilinear Kakeya problem, the Kakeya problem in finite fields, and the related joints problem. One highlight presented at the workshop was the solution of the Erdös distance problem: Given a set of N points in the plane, the number of distinct distances between these points is at least $C\sqrt{N}/\log N$.
- Harmonic analysis questions motivated by several complex variables, such as the Cauchy integral and related operators on higher dimensional domains with minimally smooth boundary, and multiplier estimates for the Kohn Laplacian on forms on the sphere of $\mathbb{C}^n$.

- The interplay between martingale methods and harmonic analysis, for example to obtain sharp weighted estimates on singular integrals, including the recent solution of the long standing A_2 conjecture, that A_2-weighted bounds for Calderón Zygmund singular integrals depend in first order on the A_2 constant of the weight. Progress has been achieved by advances on martingale based operators and sharpened transfer principles between martingale estimates and classical Calderón Zygmund theory. The methods are also applicable in the study of questions in geometric measure theory, which require understanding of singular integral theory in very hostile environments such as spaces not of homogeneous type.
- Improved understanding of invariants of higher degree analytic and smooth surfaces and their singularities as they play a role in estimating analytic expressions such as oscillatory integrals, Fourier restriction maps, or measure of sublevel sets. Typically invariants are derived from the Newton polyhedron, the convex hull of points in the integer lattice representing non-vanishing Taylor coefficients of smooth functions in question, relative to appropriate coordinates.
- Progress on multilinear estimates in recent years has been due to the application of a range of novel techniques such as time frequency analysis, additive combinatorics in the form of Gowers uniformity norms and related topics and the polynomial method. These methods also bear fruit on more classical problems such as the Hilbert transform along vector fields.
- Sharp invariance properties on quasiconformal mappings have led to new understanding of function spaces in harmonic analysis.
- New applications of real and harmonic analysis to elliptic PDE's had implications ranging from understanding of divergence form operators to nonlinear elliptic operators such as the k-Hessian.
- New localization techniques in frequency and space have led to progress on linear and non-linear dispersive PDE's.
- Harmonic analysis questions in the continuous setting have turned out to be an important guide line for the understanding of discrete analogues of these questions, and have led to interesting insights about questions from analytic number theory, such as decay estimates for exponential sums, and the interplay between number theory and real and harmonic analysis.

The meeting took place in a lively and active atmosphere, and greatly benefited from the ideal environment at Oberwolfach. It was attended by 48 participants. The program consisted of 28 lectures of 40 minutes. Long afternoon breaks have been intensively used by the participants for mathematical discussions and collaborations. The organisers made an effort to include young mathematicians, and greatly appreciate the support through the Oberwolfach Leibniz Graduate Students Program, which allowed to invite several outstanding young scientists.

Workshop: Real Analysis, Harmonic Analysis and Applications

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