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Random Matrices, Geometric Functional Analysis and Algorithms

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Abstract. The workshop gathered close to 50 participants on the topics of random matrix theory, high dimensional convex geometry and probabilistic methods in theoretical computer science. It favored cooperation between researchers in these interlaced areas by providing an interacting working atmosphere most appreciated by the participants. We hope this workshop will advance the developments at these interfaces further.

Mathematics Subject Classification (2000): 15A52, 46B07, 68W20.

Introduction by the Organisers

The workshop was successful in bringing together experts in three related fields: random matrix theory, geometric functional analysis and theoretical computer science. It was the opportunity to encourage further cooperation between people within and across these areas. 28 talks were delivered during the conference. Special efforts were made by the speakers from the three different communities to make their works accessible by the others in order to favor exchanges and discussions.

Among the specific areas discussed during the workshop, random matrix developments took an important part, as one of the hot topics of the current research. R. Vershynin presented new important results on invertibility of symmetric matrices based on Littlewood-Offord problems for quadratic forms. His talk emphasized the connection between the areas covered by the workshop, with problems coming from the random matrix theory, and methods coming from geometric functional

analysis. This number-theoretic aspect was further developed by K. Costello linking probabilistic and arithmetic properties of homogenous polynomials. V. Vu and J. Yin presented some of the most striking recent achievements on universality of the eigenvalue spacing distribution (works of T. Tao and V. Vu on one side and L. Erdös, H. T. Yau and collaborators on the other). Sharp bounds for singular values for matrices in log-concave ensembles with applications to approximate reconstruction was another highlight (talks by R. Latała, A. Litvak and N. Tomczak-Jaegermann). This series of works applies delicate geometric properties of log-concave measures to random matrices, and finds further application in signal reconstruction. M. Krishnapour presented a joint work with A. Guionnet and O. Zeitouni proving a long-standing single ring conjecture. This conjecture asserted that the empirical spectra of a unitary invariant ensemble of matrices converges to a measure, whose support is one ring, regardless of the potential. Other talks on random matrices included polynomials, tail bounds on sums of random matrices, heavy tail models, log-gases, etc. Connections between random matrix theory and quantum information theory, free probability and statistics completed the picture.

Recent developments in high dimensional convex geometry included results on tight embeddings in non-Euclidean spaces. Concentration inequalities and sharp bounds on log-concave measures with geometric applications to log-concave ensembles were presented by O. Guédon and R. Latała. B. Klartag presented a result on the vector in subspace problem which gives an application of convex geometry to the computer science area of communication complexity.

Advances in theoretical computer science are intimately related to both random matrices and high dimensional convex geometry. A talk of N. Srivastava described how a solution of a computer science problem of graph sparsification led to an unexpected improvement of several old results in geometric functional analysis. Algorithms and complexity theory figured in the talk of S. Khot on games with strong soundness, and in the talk of the structure of Y. Xiao on local optima. A talk by A. Barvinok discussed arithmetic properties of random matrices with prescribed row and column sums. Such matrices, called random contingency tables, appear in statistical analysis of large data arrays.