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Mini-Workshop: Mathematical Models, Analysis, and Numerical Methods for Dynamic Fracture

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April 24th – April 30th, 2011

Abstract. The mathematical foundation of fracture mechanics has seen considerable advances in the last fifteen years. While this progress has been substantial, it has been largely limited to quasi-static evolutions based on global energy minimization, which is known to produce non-physical results. What is missing is a generally accepted mathematical theory of dynamic crack growth, which accounts for material inertia. Such a theory would not only be able to describe the most physically realistic setting, but it would also provide a trusted starting point to resolve pressing questions about quasi-static evolutions, e.g., a rigorous justification of the quasi-static setting as an asymptotic limit of inertial dynamics. This workshop brought together researchers in mathematical analysis, mechanics, applied mathematics, and numerical analysis and laid the groundwork for progress on these questions.

Mathematics Subject Classification (2000): 74R10, 74R15, 74H20.

Introduction by the Organisers

The mini-workshop *Mathematical Models, Analysis, and Numerical Methods*, organised by Gianni Dal Maso (Trieste), Christopher J. Larsen (Worcester), and Christoph Ortner (Oxford) was held April 24th–April 30th, 2011. It was attended by 15 participants representing a broad range of expertise.

The mathematical foundations of fracture mechanics have seen considerable advances in the last fifteen years. While mathematical modelling of fracture has been a serious scientific discipline at least since the pioneering work of Griffith, there had not been a mathematically well-posed model for the prediction of crack paths until new formulations were proposed, using the framework of the calculus of variations in the function spaces BV and SBV. While there had been much previous progress, and success, in engineering models of fracture, it is the extra assumptions (e.g., regularity) that often prevent rigorous mathematical analysis. Aside from establishing a rigorous mathematical theory, the main achievement of this recent research effort was the creation of models of fracture that surpass all previous models in their flexibility of predicting crack paths.

While this progress has been substantial, it has been largely limited to quasi-static evolutions based on global energy minimization, which is known to produce non-physical results. What has been missing is a generally accepted mathematical theory of dynamic crack growth, which accounts for inertia. Such a theory would not only be able to describe the physically most realistic setting, but it would also provide a trusted starting point to resolve pressing questions about quasi-static evolutions, in particular, to rigorously justify the quasi-static setting as an asymptotic limit of inertial dynamics.

This mini-workshop brought together mathematical analysts, numerical analysts, mechanicians and applied mathematicians with expertise in fracture mechanics. Key issues in modelling, analysis, and simulation of dynamic fracture were identified. Specifically:

- Fundamental experimental results in both quasi-static and dynamic settings, which will serve as benchmarks for models and their simulation
- Comparison of energy minimization and the principle of local symmetry
- Recent progress and fundamental open problems in the mathematical analysis of quasi-static fracture
- Modeling of the free fracture surface
- Existence and uniqueness results for the wave equation off a growing straight crack, and formulas for exact solutions, in both quasi-statics and dynamics
- Issues in applying X-FEM to dynamic fracture
- Comparison of simplified 1-D dynamic models with quasi-static models
- Survey of energy release rates and their relation to Griffith fracture