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Classical and Quantum Mechanical Models of Many-Particle Systems

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Abstract. The topic of this meeting were non-linear partial differential and integro-differential equations (in particular kinetic equations and their macroscopic/fluid-dynamical limits) modeling the dynamics of many-particle systems with applications in physics, engineering, and mathematical biology. Typical questions of interest were the derivation of macro-models from micro-models, the mathematical analysis (well-posedness, stability, asymptotic behavior of solutions), and "to a lesser extent" numerical aspects of such equations. A highlight of this meeting was a mini-course on the recent mathematical theory of Landau damping.

Mathematics Subject Classification (2000): 35Qxx, 82Cxx, 82B40, 81S30.

Introduction by the Organisers

The Oberwolfach meeting described here aimed at presenting the latest mathematical results in the field of kinetic theory (both related to classical mechanics and quantum mechanics). There were 50 participants, among which 14 young participants (PhD students, post-docs or young assistant professors). Three of them (G. Raoul, E. Dolera, G. Aki) were partially sponsored by the program "Oberwolfach Leibnitz Graduate Student", and two others (C. Sparber and R. Strain) were invited within the program "US Junior Oberwolfach Fellows": they are promising young researchers working in the US.

About half of the participants gave a presentation, where the length of talks were either 30 or 45 minutes. The longer talks included an expository introduction to the subject and the shorter talks were rather on specialized results. One of the

highlights of the meeting was the presentation by C. Mouhot (in three one-hour lectures) of the remarkable theorem obtained by C. Villani and himself about the Landau damping (one of two works which were considered for awarding to C. Villani the Fields medal): the main steps of the proof were presented at this occasion, together with many important technical aspects.

Two talks (S. Mischler on one side, and F. Salvarani on the other side) are directly related to hypocoercivity, the other concept which was celebrated when the Fields medal was awarded to C. Villani.

New results on long-standing problems were reported in the talks on the derivation of kinetic equations (two on the Lorentz gas model, by E. Caglioti and B. Wennberg, still another one by M. Pulvirenti on nonlinear kinetic equations), and in the talk by R. Strain on perturbative solutions of the singular Boltzmann equation. Dolera presented his work with Regazzini in which it is proved for Maxwellian molecules under only a fourth moment hypothesis and a very mild smoothness condition that solutions of the spatially homogenous Boltzmann equation converge to equilibrium exponentially fast at a rate given by the spectral gap, even for large initial data. This had been proved earlier for interactions harder than Maxwellian by Mouhot, but his analytic proof did not extend to the Maxwellian case. Dolera and Regazini use probabilistic methods.

Non-traditional applications of kinetic theory were visited, in particular in the talks by M. Bisi (application to chemistry), J. A. Carrillo (application to the collective motion of animals), M. Herty (application to supply chain models), D. Matthes (application to socio-economics), G. Raoul (application to biomechanics and chemotaxis models). E. Sonnendrücker presented a recent numerical approach (semi-Lagrangian) to the gyrokinetic model for plasmas. It shall be used to simulate the turbulent evolution in a fusion reactor, as in the ITER project. Finally, C. Schmeiser presented a proof of existence and uniqueness for traveling waves in a chemical reaction model. Those various fields show that kinetic theory (considered in a broad sense) remains, 150 years after its discovery, a powerful tool for exploring various aspects of the reality.

The quantum mechanical talks presented here covered a broad spectrum of topics: the semi-classical limit (in particular the relation of Bohmian and Wigner measures for the linear Schröedinger equation discussed by C. Sparber, and the asymptotic dynamics for nonlinear Schrödinger equations as presented by R. Carles), macroscopic quantum models (in particular global-in-time existence results for higher order, nonlinear quantum-diffusion equations, which exploit new entropy estimates obtained by A. Jüngel and D. Matthes). Relativistic and nonrelativistic gravitational Hartree models for boson stars were analyzed in the talks of A. Michelangeli and G. Aki. M. Escobedo und X. Lu discussed the quantum Boltzmann equation for bosons. The latter author found a dichotomy betweeen an oscillatory solution and (Bose-Einstein) condensation, i.e. a singular solution. C. Negulescu presented an efficient, WKB-based numerical scheme for the stationary Schrödinger equation in the highly oscillatory classical limit.

Another emerging development stems from the interplay of entropy methods for (classical) kinetic equations on the one hand and the dissipative behavior of open quantum systems on the other hand, as presented by F. Fagnola. This seems to pave the way for a "quantum entropy method" and a better understanding of quantum hypercontractivity.

A highlight involving entropy in particle systems was the talk of F. Otto, on joint work with G. Menz that solves a long-standing problem in hydrodynamic limits. The problem is to prove a logarithmic Sobolev inequality for a Gibbs measure for N non-interacting spins, coupled only by a constraint on the total spin, and to obtain a constant that is independent of N and the value of the total spin. This had been done earlier only for the case in which the single particle energy is exactly quadratic outside a compact set. Otto and Menz introduce an interesting asymmetric variant of the Brascamp-Lieb inequality, and use it to treat a much wider range of energy functions, including the standard quartic double well.

Y. Brenier described recent work on a novel variational approach to the dynamics of a system of self-gravitating particles with sticky collisions, and J. Dolbeault also discussed variational problems relating to gravitational systems, presenting a recent result obtained with Campos and Del Pino on the existence of distinct equilibrium solutions in a stellar dynamics model. Another interesting development in particle models was presented by Y. Guo who described his work with B. Pausander giving the construction of global smooth irrotational solutions for ion dynamics in a two–fluid plasma model. Finally, J.-A. Canizo presented his results (in collaboration with L. Desvillettes and K. Fellner) about non-gelation in diffusive coagulation breakup systems.