MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Teichmüller Theory

Organised by
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ABSTRACT. This is a report on the workshop on Teichmüller theory held in
Oberwolfach, from November 28 to December 4, 2010. The workshop brought
together people working in various aspects of the field, with a focus on recent
developments. The topics discussed included higher Teichmüller theory, mod-
uli spaces of flat connections, cluster algebras, quantization of Teichmüller
spaces, the dynamical aspects of the Teichmüller and Weil-Petersson geodesic
flows, the metric and the boundary theory of Teichmüller space including the
new developments on Thurston's asymmetric metric, string topology, geomet-
ric analysis on moduli spaces, and relations with three-manifold topology and
with minimal surface theory were also highlighted. The mapping class group
was also discussed in detail, from various points of view, including its ac-
tions on simplicial complexes and on infinite-dimensional Teichmüller spaces,
its asymptotic dimension, the relation with the arc operad, the generaliza-
tions of the Johnson homomorphisms to the monoid of homology cylinders,
making contact with knot theory and with the Casson invariant and other
3-manifolds invariants. There was an open problem session, which is also
reported on here.

Introduction by the Organisers

Teichmüller theory is a subject that has multiple facets, and it is developing
in several directions with interesting interactions in low-dimensional topology, al-
gebraic topology, hyperbolic geometry, representations of discrete groups in Lie

groups, topological quantum field theory, string theory, arithmetic groups, and in many other fields.

A workshop on this subject, organised by Shygeyuki Morita (Tokyo), Athanase Papadopoulos (Strasbourg) and Robert Penner (Aarhus), was held November 28–December 4, 2010 at Oberwolfach. The topics discussed were Teichmüller theory in a broad sense, including geometric structures, quantization, cluster algebras, higher Teichmüller theory, Kleinian groups, the relation with arithmetic groups, the study of the mapping class group with its various actions and algebraic properties and the relation with 3-manifolds invariants.

The fact that Teichmüller theory makes connections with several areas in mathematics follows in part from the diversity and the richness of the structures that Teichmüller space itself carries, and classical material regarding these structures (Weil-Petersson and Teichmüller metrics, the boundary structure, etc.) was also highlighted.

This workshop was the second one held on the subject at Oberwolfach. The first one took place on May 28–June 3, 2006, with the same organizers. The number of people in the world working in the field has become relatively large, and not more than the third of the participants of the second workshop were already present of the first one. It was the intention of the organizers to include many new participants in the second workshop. The comparison between the two workshops shows that the subject is growing at an exceptional rate, and that many new ideas connections between several domains of mathematics have emerged recently.

The workshop was well attended by 53 participants with broad geographic representation from Europe, Asia and America. Many of the world specialists of the subject were present, and the organisers also kept a balance between young researchers (graduate students, post-docs) and newcomers to the field on the one hand, and senior researchers on the other hand.

The abstract of the lectures that follow provide an excellent image of the developments in the field. There was an open problem session, which is also reported on here.