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Nonlinear Waves and Dispersive Equations

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Abstract. The aim of the workshop was to discuss current developments in nonlinear waves and dispersive equations from a PDE based view. The talks centered around rough initial data, long time and global existence, perturbations of special solutions, and applications.

Mathematics Subject Classification (2000): 35xx.

Introduction by the Organisers

Nonlinear dispersive equations are models for nonlinear waves in a wide range of physical contexts. They display the competing or cooperating effects of linear dispersion and nonlinear interactions, which may be focussing or defocussing. They are linked to diverse areas of mathematics and physics, ranging from nonlinear optics over Fourier analysis to integrable systems.

Despite a huge range of different dispersive equations there are a number of recurrent themes. Current research aims at a conceptual understanding of phenomena across different classes of equations, and a detailed understanding of features of classes of solutions. The following themes were covered.

(1) *Rough initial data* Scaling often determines a critical Hilbert space of initial data. Wellposedness may or may not hold up to the critical space. During the last years an almost complete understanding of wave maps in 2 space dimensions and of critical nonlinear Schrödinger equations with power nonlinearities has been gained in $\mathbb{R}^n$. With these insights the Schrödinger maps are intensively studied. The situation is different for the Korteweg-de-Vries hierarchy and the Nonlinear Schrödinger equation, where specific

results throw some light upon the gap between scaling and wellposedness. A conceptual understanding of this gap seems to be out of reach. The insights gained in recent years allowed to attack global wellposedness in a number of surprising directions, including global wellposedness for a quintic nonlinear Schrödinger equation on a compact three dimensional manifold, global existence for supercritical 'stochastic' initial data and global existence for several problems from physics.

(2) *Dynamics near solitons* What happens to solitons in perturbated media? What can one say about solutions near solitons? The set of solitons appears to be surprising stable. Can one describe the interaction between modal parameters and a 'wave' part of the solution? A particular important case is blow-up along the set of solitons. The study of this mechanism gave insights into blow-up dynamics.

(3) *Applications and relations to different fields* There are prominent examples of dispersive integrable PDEs, including Korteweg-de-Vries, Nonlinear Schrödinger and Camassa-Holmes. They are asymptotic equations for example for waterwaves models. The Einstein equations in general relativity naturally lead to linear and nonlinear hyperbolic equations. There is considerable progress on the understanding of decay for linear wave equations in the Kerr geometry- potentially an important step towards stability of the Kerr family.

There is a large number of promising young mathematicians working in this area. As in previous meetings the organizers gave a strong preference to talks by young researchers.