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Mini-Workshop: Exploiting Symmetry in Optimization

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Abstract. The feasible regions of mathematical optimization models quite often exhibit a high degree of symmetry. In recent years, several groups of researchers have independently worked on algorithmic approaches to exploit such symmetries in a variety of contexts. Many of the techniques that have been developed are related or rely on similar computational tools. The workshop brought together researchers working on symmetry aspects in different areas of optimization. The exchange of state-of-the-art knowledge between these areas lead to identification of important directions for future research activities.

Mathematics Subject Classification (2000): 90Cxx, 52B15, 52B55, 05E18, 20B40.

Introduction by the Organisers

If a mathematical optimization problem has many symmetries, then algorithms need to exploit this in order to be efficient. Here, *symmetry* usually means that a (large) group acts on the set of feasible solutions with the property that the objective function of the optimization problem is constant on every orbit of the action. Such symmetries can be inherent to the problem (e.g., automorphisms of some graph on which the problem is defined), but they can also be introduced by the way the problem is modeled.

Different strategies to cope with symmetric optimization problems have been developed over the last few years. One important approach is to break the symmetry of the solution space during a branch-and-bound algorithm by trying to avoid dynamically to enter equivalent copies of parts of the search space (e.g. by *isomorphism pruning* or by *orbital branching*). Another strategy is to enhance the model

by additional constraints, preferably leaving only one representative out of every orbit of equivalent solutions. In case of the very important class of linear problems, the underlying geometric objects are convex polyhedra, whose algorithmic treatment (e.g., *polytope conversion*) in many cases needs sophisticated exploitation of symmetry as well. Quite often, the approaches heavily rely on efficient algorithms in computational group theory.

The aim of the workshop was to bring together researchers who are working on aspects of symmetry in areas such as mixed integer linear optimization, mixed integer and continuous nonlinear optimization (in particular semidefinite programming), algorithmic polytope theory, constraint programming, and computational group theory. The 17 participants, including four organizers, very well covered these topics. From the first day on the atmosphere of the workshop was very lively, the audience was actively participating in the presentations, and a lot of discussions were going on in smaller groups whenever there were no talks scheduled. In particular, there was a lot of exchange between researchers from the different communities. The doctoral students attending the workshop were very well integrated into these activities.

During the workshop, all participants and organizers gave presentations. The schedule was as follows. On Monday, there were two tutorials on computational group theory and the software package GAP, as well as three talks on research projects that aim at providing other tools for computational algebra or strongly rely on the use of GAP. The entire Tuesday was devoted to symmetries in integer linear optimization. The topics on Wednesday were polyhedral symmetries and exploitation of symmetries in computing Hilbert bases. On Thursday, nonlinear optimization as well as constraint programming were at the focus of the talks. The program on Friday started with two talks scheduled additionally upon special request of the participants. One of these talks was an online demonstration on how to use GAP to answer some questions that came up during an earlier talk. The program ended with an extensive discussion of open problems and future perspectives. Next to a lot of rather concrete requests with respect to integrating additional features into existing algebraic software tools, further research topics emerged from the discussion. Among those, the question for a detailed study to identify the symmetries arising in the most common libraries of optimization problems, new methods and paradigms for both automatically detecting symmetries in models as well as providing interfaces to the model creating user for enhancing models by information on symmetry, the continued development of alternative approaches, e.g., based on invariant subspaces or fundamental domains, and, just to name one more topic, investigations of symmetric triangulations and algorithms to construct them.

In view of the very stimulating interaction between the researchers during the workshop and of the presumably ongoing strong interest in symmetries in optimization problems (as demonstrated by the many open directions of future research), the workshop participants strongly agreed that a similar meeting in one or two years would be most desirable.