

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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New Directions in Simulation, Control and Analysis for Interfaces and Free Boundaries

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ABSTRACT. The field of mathematical and numerical analysis of systems of nonlinear partial differential equations involving interfaces and free boundaries is a flourishing area of research. Many such systems arise from mathematical models in material science, fluid dynamics and biology, for example phase separation in alloys, epitaxial growth, dynamics of multiphase fluids, evolution of cell membranes and in industrial processes such as crystal growth. The governing equations for the dynamics of the interfaces in many of these applications involve surface tension expressed in terms of the mean curvature and a driving force. Here the forcing terms depend on variables that are solutions of additional partial differential equations which hold either on the interface itself or in the surrounding bulk regions. Often in applications of these mathematical models, suitable performance indices and appropriate control actions have to be specified. Mathematically this leads to optimization problems with partial differential equation constraints including free boundaries. Because of the maturity of the field of computational free boundary problems it is now timely to consider such control problems.

In order to carry out design, control and simulation of such problems interaction is required between distinct mathematical fields such as analysis, modeling, computation and optimization. By bringing together leading experts and young researchers from these separate fields we intended to develop novel research directions in applied and computational mathematics. The aim of the workshop here was to focus on emerging new themes and developments in these fields and to establish and extend links between them.

Mathematics Subject Classification (2000): MSC: 35-XX, 49-XX, 65-XX IMU: 11, 16, 17.

Introduction by the Organisers

The meeting was attended by 53 participants from Austria, Belgium, China, France, Germany, Great Britain, Japan, Portugal, Spain and the United States, with expertise from three main areas: optimal control of partial differential equations, modeling involving free boundary problems and mathematical and numerical analysis of free boundary problems. Apart from discussing current problems, techniques and issues across the differing communities the focus of the workshop was set on developing the necessary analytical and numerical techniques required to successfully tackle new emerging classes of problems related to the following themes:

- (1) Computational and analytical approaches to interfaces and free boundaries,
- (2) Control and optimization of interfaces and free boundaries,
- (3) Numerical treatment and control of surface partial differential equations.

The presentations of Abels, Asai, Bellettini, Bothe, Garcke, Giga, Kohsaka, Niethammer, Röger and Santosa concerned analytical approaches to interfaces and free boundaries. While Feng, Kornhuber, Nürnberg, Pozzi, Reusken, A. Schmidt, Stoll, and Tobiska gave talks on numerical approaches to interfaces and free boundaries. Control and optimization with focus on interfaces and free boundaries was the subject of the talks of Bernauer, Casas, Deckelnick, Günther, Hintermüller, Raymond, S. Schmidt, Siebert, Vierling and Yan. Elliott gave a survey talk on the treatment of surface partial differential equations with surface finite elements, and Voigt, in his talk, introduced a diffuse-interface approach for the numerical treatment of coupled bulk/surface partial differential equations. Finally, Sprekels, in his talk, reported on the state of the art of mathematical and technical achievements in Czochralski crystal growth. This method may be considered a model application containing many of the topics considered within the workshop.

To offer young researchers a stage for presenting their research, a young researcher session was organized on Wednesday evening where the Heizaemon Honda Scholar Asai together with the Oberwolfach Leibniz Graduate Students Bernauer, Günther, S. Schmidt, and Vierling took this opportunity and gave talks on their current research results.

Surveys and articles concerning mathematical and numerical approaches to interfaces and free boundary problems may be found in the conference proceedings [5, 6, 2, 8]. The level set approach to related problems of optimal design are surveyed in [3]. The book [1] contains theoretical results for optimal control of variational inequalities. Modern mathematical concepts of control and optimization with partial differential equation constraints are developed in the book [7]. Also we mention a survey of numerical methods for interface evolution involving curvature, [4]. Finally we remark that many recent references concerning the issues of the workshop are provided at the end of the each extended abstract.

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