

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mini-Workshop: Valuations and Integral Geometry

Organised by
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ABSTRACT. As a generalization of the notion of measure, valuations have long played a central role in the integral geometry of convex sets. In recent years there has been a series of striking developments. Several examples were presented at this meeting, e.g. the work of Bernig and Fu on the integral geometry of groups acting transitively on the unit sphere, that of Hug and Schneider on kinematic and Crofton formulas for tensor valued valuations and a series of results by Ludwig and Reitzner on classifications of affine invariant notions of surface areas and of convex body valued valuations.

Mathematics Subject Classification (2000): 52B45, 52A22, 53C65.

Introduction by the Organisers

The workshop *Valuations and Integral Geometry*, organized by Semyon Alesker, Andreas Bernig, and Franz Schuster was held from January 17th to January 23rd, 2010. The meeting was attended by 16 participants working in different areas such as convex and differential geometry or geometric measure theory. The program involved 3 lecture series by Fu, Ludwig, and Reitzner as well as several one hour and shorter lectures built around them. Some highlights of the program will be described in the following.

In a 3 hour lecture series, Joseph Fu presented intriguing recent results about the product and convolution structures on the space of continuous translation invariant valuations. This new algebraic machinery has been the key tool for a fuller understanding of the kinematic formulas for groups acting transitively on the unit sphere obtained by Alesker, Bernig, and Fu. Fu described in detail

the hermitian case and presented a mysterious conjecture concerning the integral geometry in complex space forms.

In a related vein, Judit Abardia presented her joint work with Eduardo Gallego and Gil Solanes on the integral geometry in complex space forms. Some starting points for an integral geometry in Hermitian symmetric spaces were discussed by Hiroyuki Tasaki. Daniel Hug gave a very clear talk about kinematic formulas for tensor valuations which he recently obtained in a joint work with Rolf Schneider. Rolf Schneider talked about zonoids and Crofton formulas in Minkowski spaces, and Wolfgang Weil about translative kinematic formulas. Closely related to these developments is a generalization of the notion of valuations to smooth manifolds which was explained by Semyon Alesker in an impromptu evening lecture.

On a different line of research, Monika Ludwig gave an extremely interesting 3 hour lecture series about her characterizations of convex body valued valuations compatible with affine transformations. These results are deeply connected with the theory of isoperimetric inequalities. Here, the valuation point of view has shed new light on some classical affine isoperimetric inequalities which were shown to hold for larger classes of valuations by Haberl and Schuster. These inequalities have led to new affine L_p Sobolev inequalities and an affine symmetrization principle presented in a one hour lecture by Christoph Haberl.

Matthias Reitzner gave a 3 hour lecture series on his joint work with Monika Ludwig concerning their breakthrough in the characterization of upper semicontinuous $SL(n)$ invariant valuations. Their results classified both of the classical $SL(n)$ invariant notions of affine surface area - affine surface area and centro-affine surface area - which date back to Blaschke's school of affine differential geometry. In fact, all of the L_p affine surface areas introduced by Lutwak in early 1990's were completely characterized.

The program of the workshop also involved several excellent talks by young researchers. Thomas Wannerer spoke about his extension of Ludwig's characterization of the projection operator, one of the key concepts introduced by Minkowski for the study of projections of convex bodies. Andy Tsang gave a talk about his work on valuations defined on L_p function spaces, which presents a particularly exciting new area in the theory of valuations. Gautier Berck presented joint work with Juan-Carlos Álvarez on the use of Crofton formulas in the metrisability problem on Finsler manifolds. Gil Solanes spoke about Crofton and Gauss-Bonnet formulas which are invariant under the action of the Möbius group.