Mathematisches Forschungsinstitut Oberwolfach

Report No. 01/2010

DOI: 10.4171/OWR/2010/01

Model Theory: Around Valued Fields and Dependent Theories

Organised by
Andreas Baudisch, Berlin
Anand Pillay, Leeds
Katrin Tent, Münster
Martin Ziegler, Freiburg

January 3rd – January 9th, 2010

Abstract. The general topic of the meeting was "Valued fields and related structures". It included both applications of model theory, as well as so-called "pure" model theory: the classification of first order structures using new techniques extending those developed in stable theories.

Mathematics Subject Classification (2000): 03Cxx, 03C45, 12J10, 12J25 .

Introduction by the Organisers

The general topic of the meeting was "Valued fields and related structures". It included both applications of model theory, as well as so-called "pure" model theory: the classification of first order structures using new techniques extending those developed in stable theories.

The interactions of "theory" and "applications" were very visible in the meeting and in the list of participants which included people working in pure model theory, in more applied model theory, and researchers outside model theory hoping to use the machinery developed here. There were 21 long talks of 50 minutes each complemented by an afternoon with four 30 minute talks. The organizers are grateful to Rémy and Thuillier for giving a short course on Berkovich spaces as well as being available for a very active question session in the evening. This provided background for the tutorial given by Hrushovski and Loeser on their recent work analyzing the type space of algebraically closed valued fields.

Valued fields, often henselian, have been, for many years, important examples to which model theory and logic can be applied. This began with work of Ax-Kochen-Ersov in the 1960's, leading to an asymptotic solution to a conjecture of Artin. In the 1980's Denef made use of the model theory of the p-adic field, together with p-adic integration, to answer a question of Serre. In the 1990's the methods were generalized to "motivic" measure and integration by Denef and Loeser (following an idea of Kontsevich) with many applications to algebraic geometry. More recently, Hrushovski and Kazhdan have developed a "geometric" theory of measure and integration in valued fields, based on a detailed analysis of the category of definable sets in algebraically closed valued fields, again with new applications. This answered a question of Kontsevich and Gromov: if X, Y are smooth d-dimensional subvarieties of a smooth projective n-dimensional variety V, with $V \subset X$ and $V \subset Y$ birationally isomorphic, then $X \times \mathbb{A}^{n-d}$ and $Y \times \mathbb{A}^{n-d}$ are birationally equivalent. These developments are also behind the identification of the space of stably dominated types as one which in special cases agrees with the Berkovich space. This might pave the way to extending the Berkovich spaces to other settings using model theoretic language.

Over the same time period there have been important developments in "abstract" or "pure" model theory, based on generalizing the powerful machinery of stability to possibly unstable first order theories. One can distinguish three strands: First, the notion of o-minimal structures was introduced, influenced both by real algebraic geometry and the notion of a strongly minimal set from stability. The main examples are expansions of the field of real numbers by certain analytic functions, as the exponential function, where the abstract theory had many applications. Recently o-minimal structures were considered as special structures with NIP (i.e. having the non-independence property). This has been enhanced by the general theory of o-minimality, related to definably compact groups and measures, and also by the general theory of forking in theories with NIP.

Furthermore, various general notions like C-minimality and P-minimality, attempting to include nice valued fields, have been formulated. This developed into a modern model theory of algebraically closed valued fields, which lead to the work of Hrushovski and Kazhdan mentioned above. The core theories of valued fields are neither o-minimal nor simple, but have the NIP, so that these new methods do apply.

Recently, these strands have lead to a new general theory of "metastability" generalizing the theory of algebraically closed valued fields. There is a wider class of theories (not having Shelah's Tree property of the second kind), which comprises both, o-minimal and simple theories. Both o-minimal as well as simple theories were intensely studied in the 1990's. These new methods start to lead to a more uniform treatment of this wider class of theories. Some new results and conjectures concerning groups definable in o-minimal and NIP theories were given by Hrushovski in his final lecture, influenced by some fascinating analogies between o-minimal theories and valued fields.

Among other related and striking contributions were: Berarducci's talk relating o-minimal and classical homotopy in the context of definably compact groups in o-minimal structures, Scanlon's somewhat conjectural talk about motivic integration and valued difference fields, and Peterzil's talk on uniform definability of generalized exponential maps in o-minimal expansions of the reals. Zilber gave a talk about Quantum Field Theory and Zariski geometries.

There were also several exciting contributions by young researchers on the NIP theories, pairs of structures and stable fields.