Mathematisches Forschungsinstitut Oberwolfach

Report No. 55/2009

DOI: 10.4171/OWR/2009/55

Mini-Workshop: Modeling and Understanding Random Hamiltonians: Beyond Monotonicity, Linearity and Independence

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December 6th – December 12th, 2009

Abstract. The mini-workshop was devoted to the spectral analysis of random Schrödinger-type operators. While this topic has been intensively studied by physicists and mathematicians for several decades, more recently there has been particular attention devoted to models where the random parameters enter the model in a non-monotone or non-linear way. Most of the established methods applied for random operators, in fact, hinge on the presence of monotonicity w. r. t. randomness. Thus the treatment of non-monotone models forces a deeper analysis of the structure of random Hamiltonians and, in particular, the interplay of the kinetic and the potential energy parts.

Mathematics Subject Classification (2000): 82B44, 81Q10, 47B80.

Introduction by the Organisers

The mini-workshop was organised by G. Stolz and I. Veselić and brought together sixteen participants from six countries. An introductory talk by one of the organizers was followed by 15 lectures of participants, including survey talks as well as lectures on results of recent research. Nevertheless, the mini-workshop format left time for intense collaborative work which was pursued in small group discussions throughout the week.

The workshop activities focused on open problems in the theory of localization of random Schrödinger operators. A central theme was to discuss difficulties which arise due to the lack of monotonicity properties in some models of random Schrödinger operators. Such monotonicity properties have been heavily exploited in many of the rigorous results obtained for the Anderson model over the last three decades. While the standard discrete and continuum Anderson models depend monotonically on the random parameters, this is not the case for other important quantum mechanical models of disordered media. Examples of such models are Anderson-type models with sign-indefinite single site potentials, models for structural disorder such as the Poisson and random displacement models or random wave guides, Schrödinger operators including random magnetic fields or random spin matrices, certain ergodic Hamiltonians with "pseudo-random" properties (induced e.g. by the skew-shift or doubling map), or random Schrödinger operators with discretely distributed random parameters such as the Bernoulli-Anderson model or Laplacians on random graphs.

A number of talks focussed on some of the central tools in localization theory where monotonicity properties (or their lack) play a significant role in proofs: Wegner estimates, fractional moment bounds, and Lifshits tail asymptotics of the integrated density of states. Some mechanisms were identified which allow to recover monotonicity properties in some of the models or replace them with other tools such as arguments involving convexity or analyticity. One main reason for the interest in these results is that they generally require a better understanding of the underlying physics, in particular, the interaction of kinetic and potential energy in the form of uncertainty principle relations. In fact, many existing proofs which use the monotone dependence of the potential on the random parameters disregard completely the properties of the kinetic energy part of the Schrödinger operator.

Most of the participants have been working in the field of random operators before, which enabled intense and flexible discussions during and after the lectures. A few experts from other fields (namely asymptotic analysis and probability theory) have been invited with the intent to provide new tools which may be used to tackle the challenges not approachable by current methods. For instance, the lecture by W. König gave insight into how probabilistic methods are used to yield a detailed analysis of the intermittency phenomenon for the parabolic Anderson model. Similar ideas may lead to a proof of localization which does not hinge on the regularity of the individual random variables. On the final day of the workshop a lecture (by W. Kirsch) was presented in a joint session with participants of the Mini-Workshop on "Geometry of Quantum Entanglement".

Summarizing, one can say that the discussions at the workshop led to a better understanding of common themes in various non-monotone models, which had previously been investigated for specific random Hamiltonians. A clearer picture arose of the difficulties due to non-monotonicity as well as how (and if) they can be remedied. In addition, a number of open challenges were identified, examples being:

- Find a localization proof for the discrete multi-dimensional Bernoulli-Anderson model. In particular: How can recent work by Bourgain and Kenig in the continuum be carried over to discrete models?
- Identify an analogue or replacement of the usual unique continuation property for solutions of *discrete* Schrödinger operators.

- Show localization at the spectral edges of Laplacians on percolation graphs in the supercritical regime.
- Find ways to understand that infinite volume quantities (such as the IDS) are “smoother” than their finite volume counterparts.
- Use multiple averaging to show that the expectation of the eigenvalue counting function has regularity beyond the one of the distribution of a single random variable.