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# Convex Geometry and its Applications

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Abstract. The geometry of convex domains in Euclidean space plays a central role in several branches of mathematics: functional and harmonic analysis, the theory of PDE, linear programming and, increasingly, in the study of other algorithms in computer science. High-dimensional geometry, both the discrete and convex branches of it, has experienced a striking series of developments in the past 10 years. Several examples were presented at this meeting, for example the work of Rudelson et al. on conjunction matrices and their relation to confidential data analysis, that of Litvak et al. on remote sensing and a series of results by Nazarov and Ryabogin et al. on Mahler's conjecture for the volume product of domains and their polars.

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## Introduction by the Organisers

The meeting *Convex Geometry and its Applications* organised by Keith Ball, Martin Henk and Monika Ludwig, was held from November 29 to December 5, 2009. The meeting was attended by some 50 participants working in all areas of high-dimensional geometry. The program involved 10 plenary lectures of one hour's duration and about 20 shorter lectures. Some highlights of the program were as follows.

Alexander Barvinok gave an extremely intriguing talk about new approximate formulae for the volume of (or number of lattice points inside) a body determined by linear programming constraints. These formulae are based on a surprising use of the central limit theorem and its refinements, and are accurate under quite weak conditions on the constraints. For example, the formulae are correct apart from a constant factor for transportation polytopes: an astonishing degree of accuracy in high-dimensional spaces.

Mark Rudelson gave a very clear talk about the amount of noise that must be added to a contingency table of confidential attributes, before the release of statistics based on the table, in order to ensure the privacy of the individuals represented. The problem is to add the minimum amount of noise that will make reconstruction of the table impossible from the statistics that are made public. This minimum amount of noise is determined by the least singular value of the conjunction matrix (formed by entry-wise multiplication of the rows of the contingency matrix). Rudelson and his collaborators were able to employ an array of machinery concerning smallest singular values, developed mainly by participants at this meeting (especially Rudelson himself).

In a related vein, Alexander Litvak spoke about estimates for the largest and smallest singular values for random matrices with columns that are independent but whose entries are not. This considerably weakens the conditions under which such estimates have been found and makes it possible to answer completely, a question of Kannan, Lovász and Simonovits on the empirical sample size needed to estimate the covariance matrix of a domain. The results also extend the range in which the remote sensing method of Candes, Donoho and Tao can be applied.

There has been a sudden upsurge in interest in the famous conjecture of Mahler that the product of the volumes of a domain and its polar should be minimised by the simplex (or the cube/cross-polytope pair in the symmetric case). Franck Barthe explained the surprisingly delicate proof of his result with Matthieu Fradelizi: that the conjecture holds for domains having many symmetries. Dmitry Ryabogin explained his recent joint work showing that the cube is a local minimiser for the volume product, among symmetric domains. He also gave an impromptu evening lecture on Nazarov's complex-analytic proof of the approximate Mahler conjecture proved by Bourgain and Milman.

Guillaume Aubrun presented his very elegant proof of the recent result of Aldaz on the unboundedness (as a function of dimension) of the weak 1-1 norm of the maximal operator for high-dimensional cubes. Aubrun's proof uses accurate probabilistic tools for counting lattice points in high-dimensional cubes and yields a stronger lower bound than the original proof.

There were several excellent talks by young researchers. Luis Rademacher presented his solution to a problem that had become well-known from the work of Bárány, Vu, Reitzner and others: is it true that if $K$ and $L$ are convex domains with $K \subset L$, then a random simplex in $K$ (a simplex with corners chosen independently at random from $K$) has smaller expected volume than a random simplex in $L$? Bizarrely, the answer is no and this helps to explain the difficulty in estimating volumes. Eugenia Saorín presented her joint solution of a problem going back to Hadwiger on the differentiability of extensions of the classical quermassintegrals. Hadwiger originally asked the question only in dimension 3 but the characterisation given here extends to higher dimensions. David Alonso-Gutiérrez spoke about his joint work on the slicing conjecture for domains with few vertices giving a simplified proof of the result of Junge in this direction and establishing a bound independent of dimension for domains whose number of vertices is proportional to dimension. Gergely Ambrus discussed the polarisation problem, which arises from the study of polynomials on normed spaces, and his remarkable solution of the 2-dimensional case of the *strong* polarisation problem, using Blaschke products. There was widespread view that the (relatively) new arrangements to support young visitors to Oberwolfach are paying off handsomely.