

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Explicit Methods in Number Theory

Organised by
Karim Belabas, Talence
Hendrik W. Lenstra, Leiden
Don B. Zagier, Bonn

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ABSTRACT. These notes contain extended abstracts on the topic of explicit methods in number theory. The range of topics includes asymptotics for field extensions and class numbers, random matrices and L -functions, rational points on curves and higher-dimensional varieties, and aspects of lattice basis reduction.

Mathematics Subject Classification (2000): 11–xx, 12–xx, 13–xx, 14–xx.

Introduction by the Organisers

The workshop *Explicit Methods in Number Theory* was organised by Karim Belabas (Talence), Hendrik W. Lenstra (Leiden), and Don B. Zagier (Bonn), and it took place July 12–18, 2009. Five previous workshops on the topic had been held in 1999, 2001, 2003, 2005 and 2007. The goal of the meeting was to present new methods and results on concrete aspects of number theory. In several cases, this included algorithmic and experimental work, but the emphasis was on the implications for number theory. There were two ‘mini-series’ highlighting important recent developments: one of three hours, by Henri Darmon, on cycles on modular varieties and on rational points on elliptic curves via a generalisation of Heegner points; and one of three hours by Bjorn Poonen, on rational points on higher-dimensional varieties and on obstructions to weak approximation and to the Hasse principle. Some of the other themes were:

- Automorphic forms
- Rational and integral points on curves and higher-dimensional varieties
- Class numbers of number fields and of other rings
- Solving specific diophantine equations

- Computations of Tate-Shafarevich groups and of Selmer groups.

As always in Oberwolfach, the atmosphere was lively and active, providing an ideal environment for the exchange of ideas and productive discussions. The meeting was well-attended, with 50 participants from a variety of backgrounds, including a large number of younger researchers. There were 35 talks of various lengths, and ample time was allotted to informal collaboration.