Algebraische Zahlentheorie

Organised by
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ABSTRACT. The workshop brought together researchers from Europe, the US and Japan, who reported on various recent developments in algebraic number theory and related fields. Dominant themes were p-adic methods, L-functions and automorphic forms but other topics covered a very wide range of algebraic number theory.

Introduction by the Organisers

The talks in this workshop gave a very broad perspective of recent developments in algebraic number theory. The topics treated can be grouped together in several dominating themes: Height pairings for cycles on Shimura varieties and derivatives of L-functions, p-adic methods (p-adic Galois representations, relative Fontaine theory and parallel transport for p-adic vector bundles), new results on Mordell-Weil groups for elliptic curves, Iwasawa theory and L-values, higher dimensional class field theory.

Three talks were related to the relation between height pairings of cohomologically trivial cycles and derivatives of L-functions. The talk by Bruinier reported on joint work with Yang about the Arakelov height pairing of cycles on the Shimura variety for the group $O(n, 2)$, where the cycles are defined by Shimura varieties for the group $O(n-1, 2)$. The talk by Zhang was devoted to his joint result with Yuan and Zhang on the relation of a height pairing of Gross-Schoen cycles on 3-fold products of Shimura curves to the derivative of the triple product L-function. Results in a similar direction were also presented by Howard (joint work with Yang). They show that the intersection numbers of Hirzebruch-Zagier cycles at

finite places are encoded Fourier coefficients of the derivative of a non-holomorphic Eisenstein series.

Another group of three talks concerned Fontaine's theory of p-adic Galois representations of local fields. This theory is extremely active, in particular in connection with the p-adic Langlands program. Berger reported on the classification of potentially trianguline representations in dimension 2, a notion introduced by Colmez in connection with his work on the p-adic Langlands correspondence. Carouso reported on two results about the ramification of semi-stable Galois representations, treating the cases of tame and wild inertia actions. Fontaine's result concerned an elaboration of results by Kisin on finite group schemes.

Werner talked about joint work with Deninger on vector bundles on p-adic curves and parallel transport. In contrast to earlier result one can now also treat vector bundles which have strongly semi-stable reduction after pullback to a ramified covering.

The talk by Andreatta was about a relative version of Fontaine's theory and the application to Faltings' comparison result.

The generalization of the ∞-fern introduced by Gouvea, Mazur and Coleman for modular curves to the Galois representations of type $U(3)$ was presented by Chenevier.

Kerz presented a new approach to higher dimensional class field theory pioniered by Wiesend, which was refined and elaborated by him in joint work with Schmidt.

The talk by Geisser was somewhat related. He discussed Suslin homology and cohomology and especially its p-part. He formulates a generalization of a conjecture by Kato and explained the relation to higher dimensional class field theory.

Stix discussed non-abelian examples of the section conjecture. He showed that certain curves admit no sections by showing that the Brauer-Manin obstruction is the only obstruction to rational points.

A new approach to Ekedahl-Oort strata via level-1-truncations of loop groups was presented by Viehmann. In fact all known relations between these strata can be expressed in terms of group theoretical data of a loop group attached to the corresponding Shimura variety of PEL-type.

The talk by Jannsen was of a more algebraic geometric nature and presented a canonical resolution of singularities of 2-dimensional excellent schemes. This very strong result is needed in a lot of arithmetic applications.

Two talks presented new results on ranks of Mordell-Weil groups of elliptic curves. Dokchitser presented the result obtained with his brother about the parity of ranks of elliptic curves. They can show, that if the Shavarevich-Tate group is finite, then the parity of the Mordell-Weil rank is completely determined by the sign of the root number. The other result, by Mazur and Rubin, is that over each number field there are infinitely many elliptic curves of Mordell-Weil rank 0 and if the dimension of the 2-torsion of the Shavarevich-Tate group is even, then there are even infinitely many curves of rank 1.

There were two talks devoted to Iwasawa theory. Kakde talked about the results in his thesis about congruences in non-commutative Iwasawa theory for totally real

fields. Building on work of Kato, he was able to prove the congruences necessary to show the Iwasawa main conjecture for some semi-direct products of abelian groups.

Van Order explained her results on the two variable main conjecture for elliptic curves over $\mathbb{Q}$ in the $\mathbb{Z}_p^2$-extension over an imaginary quadratic field. Here she obtains some divisibility results, building on work by Kato and Rohrlich.

Goncharov explained his construction of mixed motives via his theory of "Hodge correlators". The Hodge realization of these motives can be described in terms of Green functions and their derivatives. For modular curves one gets in particular the Beilinson-Kato elements.