Mathematisches Forschungsinstitut Oberwolfach

Report No. 19/2009

DOI: 10.4171/OWR/2009/19

Homotopy Theory of Function Spaces and Related Topics

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April 5th – April 11th, 2009

Abstract. This workshop brought together researchers studying a variety of problems related to the homotopy theory of function spaces. Topics covered included: evaluation maps and Gottlieb groups, the classification of gauge groups and of other function space components, algebraic models for function spaces both in the rational and in the p-local settings, operads, configuration spaces, free and based loop spaces and infinite-dimensional Lie groups.

Mathematics Subject Classification (2000): 55P48, 55P60, 55P62, 22E65.

Introduction by the Organisers

The study of function spaces from an algebraic topological point of view dates back, at least, to the 1950s. G. Whitehead posed the basic problem of classifying the path components of a function space up to homotopy type and obtained the first results on this problem as an early application of the Whitehead product. Subsequent work of Thom and Federer paved the way for the computation of algebraic invariants of function spaces.

In the late 1960s, Gottlieb initiated the study of the evaluation map, the evaluation subgroups and, in particular, the Gottlieb groups of space. In the 1970s, Hansen, Möller, Sutherland and others studied the homotopy classification problem for the components of a function space with many complete results. An early, famous application of Sullivan's rational homotopy theory, the Vigué-Sullivan model for the free loop space of a manifold, led to a solution of the closed geodesic problem and showed the power of Sullivan's algebraic models for homotopy theory.

The 1980s saw steady progress on function spaces, especially in the local settings. Following Sullivan's sketch, Haefliger described a model for the rational homotopy type of the space of sections of a nilpotent fibration. Félix, Halperin and Thomas obtained global results on the vanishing and dimension of rationalized Gottlieb groups. Brown, Peterson and L. Smith gave a second rational model for function spaces in terms of Lannes's division functor. Finally and notably, Miller published his celebrated proof of the Sullivan conjecture concerning the contractibility of certain functions spaces during this period, a major advance in homotopy theory.

In recent years, the study of function spaces and related topics has expanded and accelerated. Whitehead's original classification problem is actively researched in the context of gauge groups. Gottlieb groups remain a challenging computation problem in the integral setting and, after rationalization, are the subject of a basic conjecture in rational homotopy theory. The study of the free loop space of a manifold has undergone a renaissance with the discovery of Chas-Sullivan string topology. Meanwhile, the further development of algebraic models for the rational and p-local homotopy theory of function spaces has opened the field to whole new types of questions, computations and, significantly, applications of function space techniques in other areas of homotopy theory.

This workshop included 23 mathematicians with expertise and active research programs in these various areas. In addition to specialized talks, there were several invited survey talks on broad topics including Gottlieb groups, gauge groups and algebraic models for function spaces after localization. There were two extended problem sessions.