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Mini-Workshop: Numerics for Kinetic Equations

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ABSTRACT. Kinetic equations are crucial to an adequate description of many processes of scientific and industrial importance. In recent years there have been intensified research activities in the field of numerical algorithms for kinetic equations related to new areas of application. Typical gas flows in micro- and nanomachines are in the rarefied regime. Thus the classical Boltzmann equation is often used to model such flows. Furthermore, the inelastic Boltzmann equation describes low density flows of granular material. Finally, flows of electrically charged particles are described by semiconductor transport equations. There are significant numerical challenges related to these applications. In low Mach number rarefied flows there is a very small signal-to-noise ratio. Therefore, variance reduction techniques for the commonly used Direct Simulation Monte Carlo method are needed. On the other hand, deterministic algorithms become more competitive. The workshop brought together leading experts from various fields to discuss recent approaches addressing the numerical challenges related to the novel applications mentioned above.

Introduction by the Organisers

The workshop *Numerics for Kinetic Equations*, organized by Irene Gamba (Austin, USA), Sergej Rjasanow (Saarbrücken, Germany), and Wolfgang Wagner (Berlin, Germany) was held November 16th – November 22nd, 2008. The meeting was well attended with 15 participants (11 from Europe, 3 from USA, and 1 from Japan). This workshop brought together researchers with various backgrounds: mathematics, physics, chemistry, and engineering science. The interdisciplinarity of the invited participants was essential because the numerical solution of kinetic equations is characterized by a wide area of applications (rarefied gas dynamics, modeling of semiconductors, plasma, or combustion). Furthermore, the strong

coupling between different areas of science is needed to understand the complicated and highly nonlinear nature of kinetic equations. The last decades have shown an amazing development in the design of effective numerical methods for kinetic equations describing processes in micro- and nanomachines. A particular example is the Knudsen compressor - an energy efficient micro-scale pump with no moving parts. Furthermore, the inelastic Boltzmann equation is more and more used in applications. Finally, semiconductor transport models are still a challenge for numerics.

In this mini-workshop, we have discussed recent mathematical and computational issues, mainly related to the so called "low Mach number" flows, from the point of view of numerical efficiency and accuracy. When solving those problems by the use of the Direct Simulation Monte Carlo method, one has to deal with a small signal-to-noise ratio. Thus, due to statistical fluctuations, it is difficult to compute the physical flow parameters (density, stream velocity, temperature, and, especially, heat flux) with sufficient accuracy, so variance reduction methods are desperately needed. On the other hand, deterministic algorithms do not exhibit statistical fluctuations, and, therefore, may become competitive for slow flows.

The talks have been devoted to

- direct simulation Monte Carlo methods for kinetic equations, especially for very slow flows close to equilibrium (Aoki, Garcia, Hadjiconstantinou, Pareschi),
- modeling and numerical solution of the kinetic transport in semiconductors (Degond, Gamba, Muscato),
- deterministic numerical methods for kinetic equations (Babovsky, Filbet, Kirsch, Rjasanow, Russo),
- kinetic equations and numerics for reactive gas flows and combustion (Kraft, Vikhansky).

Furthermore, three extended "round table" discussions were devoted to

- deviational particle methods for kinetic equations,
- stochastic fluctuations in Monte Carlo methods,
- benchmark problems for numerical methods.

These discussions were extremely useful for facilitating a clear understanding of the main difficulties when applying stochastic and/or deterministic methods to kinetic equations close to the hydrodynamic limit.

The participants appreciated the idea of the mini-workshops - to have not too many talks, but instead plenty of time for intensive and detailed discussions. We would strongly support further such workshops. Finally, we would like to thank both the administration and the staff of the MFO for providing excellent working conditions and creating a pleasant atmosphere.