Mathematisches Forschungsinstitut Oberwolfach

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Mini-Workshop: Mathematical Approaches to Collective Phenomena in Large Quantum Systems

Organised by
Stefan Adams, Leipzig/Warwick
Marek Biskup, Los Angeles
Robert Seiringer, Princeton

August 31st – September 6th, 2008

Abstract. Despite enormous progress in the last couple of decades, collective phenomena still mark one of the basic challenges of mathematical statistical mechanics. A number of approaches have been developed that provide a rigorous control of classical systems; however only few are directly applicable to quantum systems and so many basic, but extremely important, quantum collective effects remain beyond the scope of present-day mathematics. The workshop brings together people of different backgorund working on collective phenomena of quantum systems.

Mathematics Subject Classification (2000): 82B10; 82B20; 82B21; 82B28; 82B26; 82C10;60G55; 60F10: 82C10; 81S40.

Introduction by the Organisers

The workshop *Mathematical Approaches to Collective Phenomena in Large Quantum Systems*, organised by Stefan Adams (Warwick) and Robert Seiringer (Princeton) was held August 31st–September 6th, 2008. This meeting was well attended with 17 participants with broad geographic representation from all continents and with a good mixture of younger Postdocs/researchers (Boland, Bru, Crawford, Schlein, and Starr) and senior researchers. This workshop was a nice blend of researchers with various backgrounds ranging from analysis, probability theory, and functional integration. The fifteen talks are focused along the three general research directions developing mathematical theories of quantum phase transitions:

(1) Dilute gas limit and nonlinear effective theories
(2) Random walk representations via Feynman-Kac formula
(3) Field theory/semiclassical representations via coherent states

Area (1) is represented with new developments in talks by Lieb and Solovej focused on the Bogoliubov theory to derive second order correction to the ground state energy and in a talk by Yngvason on rotating trapped Bose gases. New developments for quantum spin systems have been reported by Nachtergaele and Starr. A new probabilistic approach has been introduced in a talk by König which shows a modelling of the free energy in the thermodynamic limit, and future will show if this approach goes beyond the existing theory on finite temperature Bose gases. An interesting area has been summarised by Ueltschi on random permutations. This area is recently under much focus in the probability community. The contribution from the youngest participant, the PhD-student Boland, showed an interesting result concerning the Bose condensate density and the density of so-called long cycles. Long cycles appear if probability mass of finite cycles gets lost in the thermodynamic limit, an area which is studied in the combinatorics community via shape measure analysis.

The schedule of the workshop allowed intensive discussion during the afternoons including Friday. The workshop has been closed with an informal discussion/meeting on Friday evening which was attended by the majority of the participants. The workshop showed interesting new developments, and it is the hope of the organisers that there will be more interdisciplinary collaborations, in particular between the analysis and the probability community.