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# Arithmetic Algebraic Geometry

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Abstract. Arithmetic geometry lies between number theory and algebraic geometry. It deals with schemes over the rings of integers of a numberfield or also over a p-adic completion. For them one investigates geometric porperties, integral points, or the cohomology. The present workshop had a heavy emphasis on p-adic cohomologies.

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## Introduction by the Organisers

The workshop covered developments in the field in the last four years. Roughly speaking arithmetic geometry consider algebraic schemes over rings of integers of numberfields. However an important tool is to first extend the base to a p-adic completion. Although both global and local problems matter this time there was a heavy emphasis on p-adic topics.

One of them is the deformation-theory of Galois-representations, leading to a proof of Serre's conjecture. here one starts with a global Galois-representation modulo p, then lifts modulo $p^2$, etc. For the lifts one requires certain local conditions (like being unramified outside a given set of places), and the most important and difficult such conditions arise at primes dividing $p$. Here the most important tool is J.M.Fontaine's theory which relates Galois-representations to filtered Frobenius-crystals.

Another spectacular progess is the proof (by Ngo) of the fundamental lemma in the theory of automorphic representations. It postulates identities of p-adicorbital integrals and is reduced to a geometric statement about perverse sheaves on Hitchin-fibrations in positive characteristic.

Concerning p-adic cohomology theories we are getting closer to a p-adic theory of D-modules, and of overconvergent crystals, over singular schemes. Also

the long awaited etale coverings of p-adic period domains have finally been constructed, after it has been understood that they have "holes" which are visible in the Berkovich-space but not in the conventional rigid space. That they exist is suggested by Fontaine's theory. These period domains classify p-divisible groups. Some of them (Drinfeld, Lubin-Tate) can be covered by explicit affinoid domains, thus giving some type of reduction theory for p-divisible groups. There are attempts to extend this to finite flat group-schemes.

Concerning K-theory the classical Borel-regulator from K-theory to Deligne-cohomology has been extended to syntomic cohomology, as well as the computation of its values on Eisenstein-symbols. For the l-adic etale theory general finiteness theorems can now be shown for quasi-excellent schemes. A further topic was the theory of p-adic Banach-representations of p-adic Lie-groups.

On a more global level we had talks about (p-adic!) constructions of rational points on elliptic curves, the association of K-classes to abelian varieties, and the theory of tame fundamental groups. Finally the theory of small points has been extended to function fields (over numberfields it leads to equidistribution) using tropical geometry.