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Automorphic Forms, Geometry and Arithmetic

Organised by
Stephen S. Kudla, Toronto
Joachim Schwermer, Wien

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Abstract. This meeting provided an overview of recent developments in the theory of automorphic forms and automorphic representations. In addition, new results in related areas including geometry, arithmetic geometry, moduli spaces, Galois theory and number theory, often involving the application to automorphic forms, were discussed.

Mathematics Subject Classification (2000): 11Fxx, 14Cxx Secondary: 14Gxx, 14Fxx, 11Gxx.

Introduction by the Organisers

The theory of automorphic forms has its roots in the early ninteenth century in classical work of Euler, Gauss, Jacobi, Eisenstein, and others. The subject experienced a vast expansion and reformulation following the work of Selberg, Harish-Chandra, and Langlands, in the 1970's, and remains the focus of intense current activity. The goal of this meeting was two-fold, first to provide an overview of the most recent developments in the theory of automorphic forms and automorphic representations, and, second, to provide a glimpse of the many closely related topics involving geometry and arithmetic where automorphic forms play an important role. Thus, one subset of the lectures (Soudry, Waldspurger, Gan, Muic, Moeglin, and Henniart) focused on automorphic forms and automorphic representations, while a second subset ranged quite widely and included geometry (Burger), arithmetic geometry (Pink, Howard, Nekovar, Yang), moduli spaces (Rapoport, van der Geer, Görtz, Ngô), Galois theory (Savin) and L-functions (Harder, Shahidi).

Among the many fundamental insights of Langlands are the following:

(a) Automorphic representations of a given reductive group G over a number field should occur in packets (L-packets or Arthur packets), parametrized by representations of the Weil-Deligne group into the Langlands dual group

${}^LG$. A local version of this should describe the (irreducible, admissible) representations of the group $G(F)$ for any local field F.

(b) It is necessary to consider the automorphic representations of all reductive groups together and, in particular, their relations, the most important of which are predicted by the principle of functoriality.

These insights lie very deep and their complete realization is still a very distant dream. Nonetheless, they have provided a guide for much of the subsequent research in this area and a number of the most important techniques that have been brought to bear were discussed at the meeting. These included the Arthur-Selberg trace formula, fundamental lemma, local and global descent, converse theorems, local theta correspondence, and Eisenstein series.

The connections of automorphic forms with geometry and arithmetic are many and important. One such set of connections occurs in the theory of Shimura varieties. Here important topics include interpretation as moduli spaces and period domains, the arithmetic of Heegner points and their higher dimensional generalizations, including their arithmetic intersections and heights, and the structure of Shimura varieties in characteristic $p > 0$. Automorphic forms have a deep connection with the geometry of locally symmetric spaces, where, for example, the boundary behavior of cohomology classes and Eisenstein series can be applied to the study of special values of L-functions. Again, all of these aspects were discussed during the program.

The meeting revealed, once again, that the theory of automorphic forms continues to be a vibrant subject in which many exciting developments can be expected in the future.

Special event

On Friday afternoon, the Oberwolfach Prize was awarded to Ngô Bao-Chao for his work on the fundamental lemma. The award presentation, by Professor Reinhold Remmert, was followed by a Laudatio given by Michael Rapoport explaining the significance of Ngô's work and describing a basic case of the fundamental lemma. Rapoport's Laudatio is included at the end of this report. Ngô then gave a lecture in which he explained some of the fundamental ideas of his proof, for example, the use of the Hitchin fibration. In the evening, there was a festive dinner.