

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Mini-Workshop:
Projective Normality of Smooth Toric Varieties**

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August 12th – August 18th, 2007

ABSTRACT. The mini-workshop on "Projective Normality of Smooth Toric Varieties" focused on the question of whether every projective embedding of a smooth toric variety is projectively normal. Equivalently, this question asks whether every lattice point in kP is the sum of k lattice points in P when P is a smooth (lattice) polytope. The workshop consisted of morning talks on different aspects of the problem, and afternoon discussion groups where participants from a variety of different backgrounds worked on specific examples and approaches.

Mathematics Subject Classification (2000): 14M25, 52B20.

Introduction by the Organisers

The mini-workshop on *Projective normality of smooth toric varieties*, organized by Christian Haase (Berlin), Takayuki Hibi (Osaka), and Diane Maclagan (New Brunswick), was held August 12th-18th, 2007. A small group of researchers with backgrounds in combinatorics, commutative algebra, and algebraic geometry worked on the conjecture that embeddings of smooth toric varieties are projectively normal. This very basic question appears in different guises in algebraic geometry, commutative algebra, and integer programming, but specific cases also arise in additive number theory, representation theory, and statistics. See the summary by Diane Maclagan for three versions of the same question.

There were a limited number of contributed talks in the mornings, setting the theme for the afternoon working groups. Monday morning began with Diane Maclagan describing the problem, and Winfried Bruns surveying the known results in the polyhedral formulation. This was followed on Tuesday morning by Benjamin J. Howard and Hidefumi Ohsugi on special cases of normality, and an

introductory talk by Milena Hering on the geometric vanishing theorem approach to the problem. On Wednesday morning Hal Schenck described a commutative algebra approach developed on site together with Greg Smith, while Sam Payne explained the Frobenius splitting approach. The commutative algebra approach, with optimization notes, continued in the talk of Ngô Viêt Trung on Thursday morning. Najmuddin Fakhruddin also explained his proof of the extended two-dimensional case on Thursday morning. Finally, on Friday we heard from Christian Haase and Andreas Paffenholz on some techniques for showing normality in special cases, and Francisco Santos on lattice Delaunay simplices which are potential starting points in search for a counterexample.

In the afternoons we split into working groups which then reported on their findings before dinner. These discussions continued through breaks, and in gaps between talks. The atmosphere of the group was very energetic, and we hope that the momentum generated during the meeting will continue with some of the ideas developed being pursued by the participants.

As a direct outcome of the workshop, we would like to mention

- many examples of very-ample-yet-non-normal polytopes found by Winfried Bruns,
- a joint effort of Christian Haase, Benjamin Nill, Andreas Paffenholz, and Francisco Santos to (finally) settle the ample+nef additivity question in dimension two, as well as
- a dynamic survey on projective normality and related questions to be edited by Diane Maclagan.

The organizers and participants sincerely thank the institute for providing excellent working conditions and the unique Oberwolfach spirit. We are also grateful for funding from the NSF grant supporting young US-based participants, which allowed an extra participant to attend.

In what follows we present, in addition to summaries of the talks, brief accounts on the outcome of brainstorming sessions and working groups.

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