Mathematisches Forschungsinstitut Oberwolfach

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Arithmetic and Differential Galois Groups

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Abstract. Galois theory is the study of symmetries in solution spaces of polynomial and differential equations and more generally of the relation between automorphism groups (or group schemes respectively) and the structure of algebraic and differential extensions. The lectures of this workshop were focused around the following five main topics: the absolute Galois group $G_{\mathbb{Q}}$ and the Grothendieck-Teichmüller group, étale fundamental groups and the anabelian conjecture, arithmetic Galois realizations and constructive Langlands program, local and global differential modules and the p-curvature conjecture, as well as Galois theory for nonlinear and partial differential equations.

Mathematics Subject Classification (2000): 12Fxx,12Gxx,12Hxx, (13Nxx,14Lxx,34Gxx).

Introduction by the Organisers

Galois theory is the study of symmetries in solution spaces of polynomial and differential equations and more generally of the relation between automorphism groups (or group schemes) and the structure of algebraic and differential extensions.

Many of the important problems in this area are connected to the classification of (algebraic or differential) Galois extensions and the study of the respective fundamental groups, e. g. via inverse problems. Other interesting points are the direct problem, i. e., the computation of (ordinary or differential) Galois groups, and related constructive aspects.

This workshop gave an overview on some important new results and developments. A second goal was the discussion of ideas for proving some open questions and conjectures as well as of new directions of research.

The main topics of the workshop were:

- The absolute Galois group $G_{\mathbb{Q}}$ and the Grothendieck-Teichmüller group,
- Etale fundamental groups and the anabelian conjecture,
- Arithmetic Galois realizations and constructive Langlands program,
- Local and global differential modules and the p-curvature conjecture,
- Galois theory for nonlinear and partial differential equations.

Besides this main program we had some reports on related subjects like the Cohen-Lenstra heuristics for number fields by J. Klüners and the behaviour of the Tate-Shafarevich group in anticyclotomic field extensions by M. Çiperiani.

It is always difficult to emphasise highlights or unexpected results, but one of them might be the result of F. Pop that henselian fields always are large, which is of great interest in field arithmetic and inverse Galois theory. Another striking result is the existence of G_2-motives over $\mathbb{Q}$ shown by M. Dettweiler which gives a positive answer to an old question of J.-P. Serre. A fruit of common effort in research in arithmetic and differential Galois theory is the development of patching methods for differential modules by D. Harbater and J. Hartmann. This method will allow yet unexpected applications in inverse differential Galois theory and other areas, like K-theory. The talk of B. Malgrange gave a vision of Galois theory for nonlinear differential equations. Exploitation of his work for special types of equations, like Painlevé equations, and the generalization of his ideas to non algebraically closed fields of constants as well as to positive characteristic will keep researchers busy for years.

Surely, many other results presented at the workshop should be pointed out here, too, like M. Raynaud's work on fundamental groups in positive characteristic, D. Bertrand's result on Schanuel's conjecture or Ch. Hardouin's generalization of q-difference equations to roots of unity by creating an iterative q-difference theory.

Altogether, we had a wonderful and inspiring week with lots of interesting lectures and many discussions bearing ideas for future research.

Finally, the organisers want to cordially thank the Oberwolfach administration and its staff for giving us the opportunity to arrange this and earlier workshops on Galois theory as well as for the excellent service.