

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Mini-Workshop: Multiscale and Variational Methods in
Material Science and Quantum Theory of Solids**

Organised by
Isabelle Catto (Paris)
Isaac Chenchiah (Bristol)
Ivan Veselić (Chemnitz)
Johannes Zimmer (Bath)

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ABSTRACT. This workshop brought together 18 scientists from three different mathematical communities: (i) random Schrödinger operators, (ii) quantum mechanics of interacting atoms, and (iii) mathematical materials science. Several underlying themes were identified and addressed: variational principles, homogenisation techniques, thermodynamic limits, spectral theory, and dynamic and stochastic aspects.

Mathematics Subject Classification (2000): 35, 49, 70, 74, 82.

Introduction by the Organisers

This workshop brought together people working in probability theory, multiscale analysis, calculus of variations and spectral theory. The purpose was to stimulate exchanges between the respective communities, and identify open problems at the boundaries and intersections of these areas.

There are several mathematical links between the theory of random Schrödinger operators, quantum mechanics of interacting atoms, and mathematical materials science. For instance, spectral analysis lies at the heart of the study of random Schrödinger operators; it is used in the quantum mechanics of interacting atoms as a natural and important tool for the analysis of Euler-Lagrange equations and their solutions; it was shown in the workshop that spectral analysis plays an important rôle in the study of photonic crystal fibres, e.g., in the study of band gaps.

Other tools of scale-bridging were discussed including homogenisation and the computation of the thermodynamic limits of various classical and quantum systems.

Several expository morning lectures explained the fundamentals of these fields to the varied audience, particularly to those in the other communities. These lectures were intended to last 60 – 75 minutes but almost always lasted 90 minutes due to lively discussions, both during and after the talks. They were complemented by specialised talks in the afternoon.

The organisers feel that this attempt to bring these different communities together was largely successful, and may have built new bridges between analysis and probability; the credit for this belonging, for the most part, to the speakers. That mini-workshops on “Levy Processes and Related Topics in Modelling” and “Control of Free Boundaries” were being held in parallel afforded many opportunities for further interactions.

There was roughly the same number of analysts and probabilists among the participants. France, Germany, the UK and the US were about equally represented; there was one participant from Denmark. Almost all participants were relatively early in their careers.

The excellent living and working conditions provided by the institute contributed to a lively scientific atmosphere. The organisers thank the NSF for funding the participation of the three speakers from the US, Patrick Dondl for collecting the extended abstracts (and the “kleine Trommler” for livening up the excursion!).