

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Theory of Water Waves

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ABSTRACT. The water-wave problem is the study of the two- and three-dimensional irrotational flow of a perfect fluid bounded above by a free surface subject to the forces of gravity and surface tension. It is a paradigm for most modern methods in nonlinear functional analysis and nonlinear dispersive wave theory. Its mathematical study calls upon many different approaches, as iteration methods, bifurcation theory, dynamical systems theory, complex variable methods, PDE methods, the calculus of variations, positive operator theory, topological degree theory, KAM theory, and symplectic geometry.

Mathematics Subject Classification (2000): 76B15.

Introduction by the Organisers

The 1990s saw vigorous activity in the mathematical theory of water waves by several independent international research groups, and in response we organised a mini-workshop in Oberwolfach in 2001 entitled *Recent Developments in the Mathematical Theory of Water Waves*. The 2001 meeting, which was devoted to the exact equations for water waves as written down by Euler, was a great success. A collection of papers originating at the meeting were published in a special issue of the *Philosophical Transactions of the Royal Society* and led to significant progress on certain famous and outstanding problems in water waves, particularly local existence and uniqueness, stability, three-dimensional waves, justification of model equations, convexity results for Stokes waves, and effective and accurate numerical schemes.

In view of the interest in water waves generated by the 2001 meeting and our subsequent endeavours, it appeared timely to bring the various research groups together in another Oberwolfach workshop to intensify research in this subject. The following topics were chosen as priority research areas for the workshop since

(i) they represent issues which have been almost fully settled for model equations, but remain almost fully open for the exact water-wave problem; and (ii) pose mathematical challenges whose resolution is likely to prove beneficial in a wide range of situations beyond the water-wave problem:

- Comparison of the diverse mathematical formulations of the hydrodynamic equations which have recently been found by different researchers;
- Numerical and analytical construction of fully localized three-dimensional solitary waves;
- Numerical and analytical investigations of stability and instability mechanisms for periodic wave trains;
- New scenarios for the generation of freak waves in long wave form;
- Construction of doubly-periodic three-dimensional water waves;
- New results for Dirichlet-Neumann operators.

Significant new results in these areas were reported at the conference and are summarised in the extended abstracts below. The workshop was attended by twenty-three researchers from eight countries; there was a good mix of researchers who had attended the 2001 meeting and those who had not, and a number of younger researchers new to this field attended. Twenty 45-minute talks were held in a friendly and informal atmosphere and, in true Oberwolfach spirit, many collaborative discussions took place.

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