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Schnelle Löser für partielle Differentialgleichungen

Organised by
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ABSTRACT. The workshop *Schnelle Löser für partielle Differentialgleichungen*, organised by Randolph E. Bank (La Jolla), Wolfgang Hackbusch(Leipzig), Gabriel Wittum (Heidelberg) was held May 22nd - May 28th, 2005. This meeting was well attended by 47 participants with broad geographic representation from 9 countries and 3 continents. This workshop was a nice blend of researchers with various backgrounds.

Introduction by the Organisers

The conference was organized by Randolph E. Bank, UCSD, La Jolla, Wolfgang Hackbusch, MPI Leipzig, and Gabriel Wittum, University of Heidelberg. This was the fourth one in a series of conferences on fast solvers held at Oberwolfach since 1999. The idea of these workshops is to bring together experts from the different thriving areas of solvers and offer a platform for scientific exchange and progress. The field of solvers for the algebraic systems arising from the discretization of partial differential equations has developed to a major area of numerical mathematics and scientific computing. Solvers are an essential part of simulation codes for problems from science and technology, in many cases determining the complexity of the whole simulation. By virtue of that, the choice of the solver can decide on the realiability of a simulation and if it can be done at all. Thus, solvers are a substantial mathematical component of most simulation tools and a major contribution of mathematics to quite a lot of applied disciplines. This has increased

the interest in mathematics of colleagues from the applied sciencies over the last decade substantially.

Major areas of solvers represented at the workshop are: Multigrid methods, H-matrices, domain decomposition methods, and conjugate gradient methods. Often these methods are combined, e.g. multigrid is mostly used as a preconditioner nowadays. Besides that, several talks were given on other aspects of solving partial differential equations, such as discretization schemes and the algebraic properties of the resulting stiffness matrices, overall solution strategies, and application areas where solving plays a crucial rôle. A total of 27 presentations gave a nice overview over the current research, open problems and new developments. Intense discussions provided the opportunity to go into details of novel algorithms and approaches.

In multigrid methods, a lot of research is going in the direction of developing robust methods for special applications. This is a challenging topic requiring mathematical expertise as well as understanding of the model and the application process itself. Another major topic is Algebraic Multigrid. AMG methods are already wide spread in several applied communities. However, a lot of open problems remains and the final algorithm is not yet in sight. Several talks also were related to performance issues of multigrid on certain computer architectures such as super scalar or parallel computers. Multigrid research is thriving more than ever.

A whole bunch of talks were about domain decomposition methods. These methods are of particular interest for multiphysics problems and parallelization issues. Several new developments have been reported and discussed, giving interesting future perspectives. Often techiques from domain decomposition analysis can be used to analyze other methods e.g. multigrid. A novel technique useful together with domain decomposition and multigrid, but can also stand on its own, are hierarchical matrices (H-matrices). Here, several talks have shown the impressive level of development these methods already have since their introduction in 1998. Further talks have discussed solver techniques for application problems e.g. low Mach-number flow or electromagnetics. as well as other problem areas like optimization. Moreover, talks about novel techniques like meshless methods and several other solver techniques have been given.

In total, the workshop was very successful in bringing together international-level experts from different areas and disciplines. Meanwhile, the Oberwolfach workshop on *Schnelle Löser für partielle Differentialgleichungen* is established as major event in the solver community and a mainstay for novel developments.