Mathematisches Forschungsinstitut Oberwolfach

Report No. 40/2004

Mini-Workshop: Compactness Problems in Interpolation Theory and Function Spaces

Organised by
Fernando Cobos (Madrid)
Thomas Kühn (Leipzig)

August 15th – August 21st, 2004

Introduction by the Organisers

Compactness is undoubtedly one of the central and most relevant notions in mathematics. The present mini-workshop was centered around some important compactness problems connected to interpolation theory and to the theory of function spaces, two very closely related areas. Our aim was to bring together leading experts and active younger researchers in these fields. The mini-workshop was attended by 16 participants from Germany (5), Spain (5), Israel (2), Poland (2), Sweden (1) and United States (1), and the main problems we dealt with can, more specifically, be grouped as follows.

1. Interpolation of compactness and related properties
   - 1.1. Complex interpolation of compact operators
   - 1.2. Real interpolation of compactness and similar properties
2. Compact embeddings in function spaces
   - 2.1. Entropy numbers of such embeddings and applications to spectral theory of differential operators
   - 2.2. Entropy techniques in sequence spaces

Let us shortly describe these topics.

1.1. An outstanding problem in interpolation theory is the question, whether the complex interpolation method preserves compactness of operators. This is open since 40 years, and by now only partial answers are known. Recently a new and promising general approach has been proposed. We discussed this approach.

1.2. For the real interpolation method, however, it is well-known that it does preserve compactness. So it is quite natural to ask for *quantitative* versions of this

purely *qualitative* result, for instance in terms of the measure of non-compactness, or in terms of entropy numbers. Another natural question is to study whether similar properties, like weak compactness, for example, are stable under real interpolation as well. These problems were addressed in some talks.

2.1. The sequence of entropy numbers $(e_k(T))_{k=1}^{\infty}$ of a bounded linear operator T between quasi-Banach spaces can be considered as a quantification of compactness, since T is compact if and only if $\lim_{k\to\infty} e_k(T) = 0$. The basis for applications to spectral theory is the famous Carl-Triebel inequality, which relates entropy numbers of Riesz operators to its eigenvalues. Many concrete problems lead to the investigation of compact embeddings of certain function spaces, e.g. Sobolev or Besov spaces. In the talks both a survey on the general framework as well as new entropy estimates for specific embeddings were given.

2.2. Using various methods, for instance wavelet or atomic decompositions, the function space embeddings can very often be reduced to embeddings of (fairly complicated) sequence spaces. For the estimation of their entropy numbers one needs many different techniques, some of them quite new. Such techniques also were the subject of talks.

Finally, several other aspects of interpolation were treated in talks, e.g. approximation spaces, bilinear interpolation, relation to eigenvalues and operator ideals. We list the abstracts of all talks in chronological order.

The scientific program started with two survey lectures by Triebel and Cwikel, leading experts in their fields, followed by two more survey-style talks of the organisers. Then all other participants reported on own recent research results. In addition to this "official" program, which was already scheduled in advance, there was a number of further activities. Several participants offered a second talk, on another topic of common interest, or continued their respective talks in order to explain some technicalities in greater detail. The remaining time was used for many intensive discussions in smaller groups, and on Friday a problem session was held. The aim was to summarize the results of the mini-workshop and to discuss and collect several relevant problems, thus pointing out possible directions for further research in our field.

Concerning social activities, one should mention the traditional hiking tour to St. Roman on Wednesday afternoon and the, maybe less traditional, joint session of all three parallel mini-workshops. The aim of this informal interdisciplinary session was to explain very briefly the kind of problems and ideas of our respective areas. Before this meeting there were serious doubts whether the intended goal would be achievable in only a few minutes, but afterwards it was general opinion that we have had a surprisingly inspiring and interesting evening, giving in fact a rough impression of the other two research areas.

Last but not least, the organisers would like to express their gratitude to the director and the authorities of the Mathematisches Forschungsinstitut Oberwolfach for making this mini-workshop possible and for the constant support in its organisation and preparation. On behalf of all participants we thank all members

of the staff for creating the unique working atmosphere, which made our stay so pleasant and which contributed substantially to the success of our mini-workshop.

MSC Classification: