Abstract

Let $s \in \mathbb{R}$, $p \in (1, \infty)$, $\tau \in [0, \frac{1}{p}]$ and $\mathcal{S}_\infty(\mathbb{R}^n)$ be the set of all Schwartz functions φ whose Fourier transforms $\widehat{\varphi}$ satisfy that $\partial^\gamma \widehat{\varphi}(0) = 0$ for all $\gamma \in (\mathbb{N} \cup \{0\})^n$. Denote by ${}_V\dot{F}^{s,\tau}_{p,p}(\mathbb{R}^n)$ the closure of $\mathcal{S}_\infty(\mathbb{R}^n)$ in the Triebel–Lizorkin-type space $\dot{F}^{s,\tau}_{p,p}(\mathbb{R}^n)$. In this paper, the authors prove that the dual space of ${}_V\dot{F}^{s,\tau}_{p,p}(\mathbb{R}^n)$ is the Triebel–Lizorkin–Hausdorff space $F\dot{H}^{-s,\tau}_{p',p'}(\mathbb{R}^n)$ via their φ-transform characterizations together with the atomic decomposition characterization of the tent space $F\dot{T}^{-s,\tau}_{p',p'}(\mathbb{R}^{n+1}_{\mathbb{Z}})$, where t' denotes the conjugate index of $t \in [1, \infty]$. This gives a generalization of the well-known duality that $(\mathrm{CMO}(\mathbb{R}^n))^* = H^1(\mathbb{R}^n)$ by taking $s = 0$, $p = 2$ and $\tau = \frac{1}{2}$. As applications, the authors obtain the Sobolev-type embedding property, the smooth atomic and molecular decomposition characterizations, boundednesses of both pseudo-differential operators and the trace operators on $F\dot{H}^{s,\tau}_{p,p}(\mathbb{R}^n)$; all of these results improve the existing conclusions.