Abstract

We investigate the point behavior of periodic functions and Schwartz distributions when the Fourier series and the conjugate series are both Abel summable at a point. In particular we show that if f is a bounded function and its Fourier series and conjugate series are Abel summable to values γ and β at the point θ_0, respectively, then the primitive of f is differentiable at θ_0, with derivative equal to γ, the conjugate function satisfies $\lim_{\theta\to\theta_0} \frac{3}{(\theta-\theta_0)^3} \int_{\theta_0}^{\theta} \tilde{f}(t)\,(\theta-t)^2\,\mathrm{d}t = \beta$, and the Fourier series and the conjugate series are both (C,κ) summable at θ_0, for any $\kappa > 0$. We show a similar result for positive measures and L^1 functions bounded from below. Since the converse of our results are valid, we therefore provide a complete characterization of simultaneous Abel summability of the Fourier and conjugate series in terms of "average point values", within the classes of positive measures and functions bounded from below. For general L^1 functions, we also give a.e. distributional interpretation of $-\frac{1}{2\pi}\mathrm{p.v.}\int_{-\pi}^{\pi} f(t+\theta_0)\cot\frac{t}{2}\,\mathrm{d}t$ as the point value of the conjugate series when viewed as a distribution.

We obtain more general results of this kind for arbitrary trigonometric series with coefficients of slow growth, i.e., periodic distributions.