

Abstract

Littlewood–Paley theory for the differential operator, $\Delta_{\mathbb{D}} = \partial_{x_1}^2 \partial_{x_2}^2 - \partial_{x_3}^2$, is developed. This study leads to the introduction of a new class of Triebel–Lizorkin spaces $\dot{F}_p^{\alpha,q}(\mathbb{D})$ associated with the dilation $(x_1, x_2, x_3) \rightarrow (2^{\nu_1}x_1, 2^{\nu_2}x_2, 2^{\nu_1+\nu_2}x_3)$, $(\nu_1, \nu_2) \in \mathbb{Z}^2$. The corresponding atomic and molecular decompositions are obtained. A frame generated by modulations, dilations and translations is also studied. Using this result, we show that $\Delta_{\mathbb{D}}$ is a linear isomorphism from $\dot{F}_p^{\alpha,q}(\mathbb{D})$ to $\dot{F}_p^{\alpha-2,q}(\mathbb{D})$.