Abstract

This paper is concerned with asymptotic behavior of generalized eigenvectors of a class of Hermitian Jacobi matrices J in the critical case. The last means that the fraction $\frac{q_n}{\lambda_n}$ generated by the diagonal entries q_n of J and its subdiagonal elements λ_n has the limit ± 2. In other words, the limit transfer matrix as $n \to \infty$ contains a Jordan box (= double root in terms of Birkhoff–Adams theory). This is the situation where the asymptotic Levinson theorem does not work and one has to elaborate more special methods for asymptotic analysis. It should be mentioned that the critical case exactly corresponds to spectral phase transition phenomena, where the spectral structure changes dramatically (from discreet spectrum to pure absolutely continuous one) whenever the parameters in matrix entries cross singular surfaces, see J. Janas and S. Naboko [Spectral properties of selfadjoint Jacobi matrices coming from birth and death processes, *Oper. Theory Adv. Appl.* 127 (2001), pp. 387–397]. A Jordan box is the limit transfer matrix for all values of the spectral parameter λ simultaneously, it describes the "moment" of spectral phase transition. An application to the case of $\lambda_n = n^\alpha(1 + r_n)$, $q_n = -2n^\alpha(1 + p_n)$ with small perturbations r_n, p_n and $\alpha \in (0, 1]$ is studied.