Abstract

A state-constrained optimal boundary control problem governed by a linear elliptic equation is considered. In order to obtain the optimality conditions for the solutions to the model problem, a Slater assumption has to be made that restricts the theory to the two-dimensional case. This difficulty is overcome by a source representation of the control and combined with a Lavrentiev type regularization. Optimality conditions for the regularized problem are derived, where the corresponding Lagrange multipliers have L^2-regularity. By the spectral theorem for compact and normal operators, the convergence result of Tröltzsch and Yousept in [*Comput. Optim. Appl.* 42 (2009), 43–66] is extended to a higher dimensional case. Moreover, the convergence for vanishing regularization parameter of the adjoint state associated with the regularized problem is shown. Finally, the uniform boundedness of the regularized Lagrange multipliers in $L^1(\Omega)$ is verified by a maximum principle argument.