Abstract

Let $X = G/H$ be a homogeneous space, $\widetilde{X} = X \times [0, \infty)$, μ a doubling measure on X induced by a Haar measure on the group G, β a positive measure on $\widetilde{X}$ and W a weight on X. Consider the maximal operator given by

$$\mathcal{M}f(x,r) = \sup_{s \geq r} \frac{1}{\mu(B(x,s))} \int_{B(x,s)} |f(y)|\, d\mu(y), \quad (x,r) \in \widetilde{X}.$$

In this paper, we obtain, for each $p, q, 1 < p \leq q < \infty$, a necessary and sufficient condition for the boundedness of the maximal operator $\mathcal{M}$ from $L^p(X, W d\mu)$ to $L^q(\widetilde{X}, d\beta)$. As an application, we obtain a necessary and sufficient condition for the boundedness of the Poisson integral of functions defined on the unit sphere S^n of the Euclidian space $\mathbb{R}^{n+1}$, from $L^p(S^n, W d\sigma)$ to $L^q(\mathbb{B}, d\nu)$, where σ is the Lebesgue measure on S^n, W is a weight on S^n and ν is a positive measure on the unit ball $\mathbb{B}$ of $\mathbb{R}^{n+1}$.