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Mackey Topologies on Vector-Valued Function Spaces

Let E be an ideal of L^0 over a σ -finite measure space (Ω, Σ, μ) , and let $(X, \|\cdot\|_X)$ be a real Banach space. Let $E(X)$ be a subspace of the space $L^0(X)$ of μ -equivalence classes of all strongly Σ -measurable functions $f : \Omega \rightarrow X$ and consisting of all those $f \in L^0(X)$ for which the scalar function $\|f(\cdot)\|_X$ belongs to E . Let $E(X)_n^\sim$ stand for the order continuous dual of $E(X)$. We examine the Mackey topology $\tau(E(X), E(X)_n^\sim)$ in case when it is locally solid. It is shown that $\tau(E(X), E(X)_n^\sim)$ is the finest Hausdorff locally convex-solid topology on $E(X)$ with the Lebesgue property. We obtain that the space $(E(X), \tau(E(X), E(X)_n^\sim))$ is complete and sequentially barreled whenever E is perfect. As an application, we obtain the Hahn-Vitali-Saks type theorem for sequences in $E(X)_n^\sim$. In particular, we consider the Mackey topology $\tau(L^\Phi(X), L^\Phi(X)_n^\sim)$ on Orlicz-Bochner spaces $L^\Phi(X)$. We show that the space $(L^\Phi(X), \tau(L^\Phi(X), L^\Phi(X)_n^\sim))$ is complete iff L^Φ is perfect. Moreover, it is shown that the Mackey topology $\tau(L^\infty(X), L^\infty(X)_n^\sim)$ is a mixed topology.

Keywords: Vector-valued function spaces, Orlicz-Bochner spaces, locally solid topologies, Lebesgue topologies, Mackey topologies, mixed topologies, sequential barreledness.

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