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Analysis of the Operator Δ^{-1}div Arising in Magnetic Models

In the context of micromagnetics the partial differential equation

$$\mathrm{div}(-\nabla u + \mathbf{m}) = 0 \text{ in } \mathbb{R}^d$$

has to be solved in the entire space for a given magnetization $\mathbf{m} : \Omega \to \mathbb{R}^d$ and $\Omega \subseteq \mathbb{R}^d$. For an L^p function $\mathbf{m}$ we show that the solution might fail to be in the classical Sobolev space $W^{1,p}(\mathbb{R}^d)$ but has to be in a Beppo-Levi class $W_1^p(\mathbb{R}^d)$. We prove unique solvability in $W_1^p(\mathbb{R}^d)$ and provide a direct ansatz to obtain u via a non-local integral operator $\mathcal{L}_p$ related to the Newtonian potential. A possible discretization to compute $\nabla(\mathcal{L}_2\mathbf{m})$ is mentioned, and it is shown how recently established matrix compression techniques using hierarchical matrices can be applied to the full matrix obtained from the discrete operator.

Keywords: Laplace equation, integral representation, Caldern-Zygmund kernel, micromagnetics, magnetic potential, panel clustering, hierarchical matrices.

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