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J. Boos, M. Zeltser

Sequences of 0's and 1's. Classes of Concrete 'big' Hahn Spaces

This paper continues the joint investigation by G. Bennett, J. Boos and T. Leiger [Studia Math. 149 (2002) 75–99] and J. Boos, T. Leiger and M. Zeltser [J. Math. Anal. Appl. 275 (2002) 883–899] of the extent to which sequence spaces are determined by the sequences of 0's and 1's that they contain. Bennett et al. proved that each subspace E of ℓ^∞ containing the sequence $e = (1, 1, \ldots)$ and the linear space bs of all sequences with bounded partial sums is a Hahn space, that is, an FK-space F contains E whenever it contains (the linear hull $\chi(E)$ of) the sequences of 0's and 1's in E. In some sense these are 'big' subspaces of ℓ^∞. Theorem 2.6, one of the main results of this paper, tells us that this result remains true if we replace bs with suitably defined spaces $bs(N)$ which are subspaces of bs when N is a finite partition of $\mathbb{N}$.
As an application of the main result, two large families of closed subspaces E of ℓ^∞ being Hahn spaces are presented: The bounded domain E of a weighted mean method (with positive weights) is a Hahn space if and only if the diagonal of the matrix defining the method is a null sequence; a similar result applies to the bounded domains of regular Nörlund methods. Since an FK-space E is a Hahn space if and only if $\chi(E)$ is a dense barrelled subspace of E, by these results, a large class of concrete closed subspaces E of ℓ^∞ such that $\chi(E)$ is a dense barrelled subspace can be identified by really simple conditions. A further application gives a negative answer to Problem 7.1 in the paper mentioned above.