Scaling Limits in Models of Statistical Mechanics

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Abstract. This conference (part of a long running series) aims to cover the interplay between probability and mathematical statistical mechanics. Specific topics addressed during the 22 talks include: Universality and critical phenomena, disordered models, Gaussian free field (GFF), random planar graphs and unimodular planar maps, reinforced random walks and non-linear $\sigma$-models, non-equilibrium dynamics. Less stress is given to topics which have running series of Oberwolfach conferences devoted to them specifically, such as random matrices or integrable models and KPZ universality class.

There were 50 participants, including 9 postdocs and graduate students, working in diverse intertwining areas of probability, statistical mechanics and mathematical physics.

Subject classification: MSC: 60,82; IMU: 10,13.

Introduction by the Organisers

This workshop was a sequel to a MFO conference, by the same organizers, which took place in 2015. More broadly, it is a sequel to MFO conferences in 2006, 2009 and 2012, organised by Ken Alexander, Marek Biskup, Remco van der Hofstad and Vladas Sidoravicius. The main focus of the conference remained on probabilistic and analytic methods of non-integrable statistical mechanics. With respect to the previous editions, greater emphasis was put on statistical mechanics models on groups and general graphs, as a lot has happened in this arena recently. The list of 50 participants reflects our attempts to maintain an optimal balance between diverse fields, leading experts and promising young researchers. Nine participants were on postdoctoral and graduate level.
In our choice of 22 talks we tried to illuminate major recent advances in the field and to expose and address at least some aspects of the works for each and every one of the participants. A more detailed account of the presentations is given below. Due to an intended intertwining of topics and themes it is hard to give an unambiguous classification.

**Statistical mechanical models on groups and general graphs.** Tom Hutchcroft described an improved proof of the Aizenman-Kesten-Newman arm exponent estimate, and applications to percolation on hyperbolic groups. Aran Raoufi described joint work with Duminil-Copin, Goswami, Severo and Yadin in which they showed that percolation on every graph with isoperimetric dimension at least 4 has a non-trivial phase transition. Christoph Garban described work on the inverted orbit of random walk on interval exchange transformation groups, related to the Thompson group.

**Two dimensional models.** The understanding of two dimensional models proceeds rapidly and we heard 5 talks on the topic.

Vincent Tassion discussed a renormalisation scheme which allows to show a quadrachotomy for the two dimensional Potts model. Béatrice de Tilière talked about massive Laplacians on isoradial graphs. Giambattista Giacomin revisited a classic paper of McCoy and Wu about analyticity of the pressure of the Ising model with columnar quenched disorder. Two more talks were on Gaussian multiplicative chaos, a model representing the scaling limit of planar random graphs: Hubert Lacoin talked about its fluctuations and Jason Miller investigated random walks and Brownian motion on it.

**Statistical mechanics models with quenched disorder.** We heard 3 talks on Processes in random environment: Marek Biskup analyzed degenerate dynamical random environment motivated by the Helffer-Sjöstrand representation. Bálint Tóth talked about the Lorentz gas with random obstacles, on time-scales beyond the Boltzmann-Grad limit. Quentin Berger studied directed polymers in heavy-tailed environment.

Two talks were devoted to spin glasses: David Belius discussed the TAP-Plefka equations for the spherical Sherrington-Kirkpatrick model, and Aukosh Jagannath discussed the Langevin dynamics and long equilibration time for mean-field generalised spin glasses.

**Other models inspired by statistical physics.** Finally, a few topics were covered by a single talk:

- Silke Rolles strengthened the connection between the vertex-reinforced jump process and the supersymmetric hyperbolic sigma model.
- Roland Bauerschmidt talked about renormalisation for hierarchical spin models and implications on the dynamical spectral gap.
- Sébastien Ott applied Ornstein-Zernike theory to study the Potts model with a defect line.
Nicholas Crawford discussed the eigenvectors of a non-Hermitian random matrix model.
Wendelin Werner talked about loop-soups and critical percolation in dimensions 7 and above.
Lisa Hartung talked about the extremal set of branching Brownian motion.
Perla Sousi talked about the capacity of the range of simple random walk in different space dimensions.
Wioletta Ruszel investigated the scaling limit of the odometer function in sandpile models.
Frank den Hollander talked about a population model with seed-bank and spatial structure.

Summary. The workshop was an obvious success. In particular, it helped to update the participants on the state of the art and on the important pending open problems in the fields related to their domain of research, facilitated exchange of ideas between researchers in technically disconnected areas, and it gave rise to many interesting and informative discussions. In particular, we had a lively evening session focused mostly on open problems. Some new collaborations arose, notably Hutchcroft and Pete solved a long-standing problem on the cost of Kazhdan groups (this was announced a few weeks after the conference ended).

We would like to thank the MFO personnel for the help and for the invaluable logistic support, as well as for creating a friendly and stimulating environment throughout the entire meeting.

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