Abstract. The aim of the series of Oberwolfach meetings on ‘Explicit methods in number theory’ is to bring together people attacking key problems in number theory via techniques involving concrete or computable descriptions. Here, number theory is interpreted broadly, including algebraic and analytic number theory, Galois theory and inverse Galois problems, arithmetic of curves and higher-dimensional varieties, zeta and \( L \)-functions and their special values, modular forms and functions.

The meeting provides a forum for presenting new methods and results on concrete aspects of number theory. Considerable attention is paid to computational issues, but the emphasis is on aspects that are of interest to the pure mathematician. In this respect the meetings differ from virtually all other computationally oriented meetings in number theory (most notably the ANTS series), which have a tendency to place actual implementations, numerical results, and cryptographic implications in the foreground.

The 2018 meeting featured a mini-course on nonabelian Chabauty theory, so several of the talks were on this topic; the other talks covered a broad range of topics in number theory.

Mathematics Subject Classification (2010): 11xx.

Introduction by the Organisers

The workshop Explicit Methods in Number Theory was organised by Karim Belabas (Talence), Bjorn Poonen (Cambridge, MA), and Fernando Rodriguez Villegas (Trieste), and it took place July 22–28, 2018. Nine previous workshops on the topic had been held since 1999. The 2018 instance of the workshop featured a mini-course on nonabelian Chabauty theory, because of the potential for advances in this field that became apparent in recent years.
The non-abelian Chabauty program is aimed at determining the rational and integral points on curves over number fields. It is a conjectural approach and though the initial theoretical ideas were put forth by Minhyong Kim over a decade ago, it was unclear for a long time whether they could ever be made practical enough to solve problems that were inaccessible by other methods. But in the last few years, new theoretical and computational methods have been developed (largely by junior mathematicians) which have begun to bear fruit — we note in particular the recent articles *Computing integral points on hyperelliptic curves using quadratic Chabauty* by Amnon Besser, Jennifer Balakrishnan, Jan Steffen Müller and *Quadratic Chabauty and rational points I: p-adic heights* by Jennifer Balakrishnan and Netan Dogra (with an appendix by Müller), the second of which determines $X(\mathbb{Q})$ for a curve $X$ for which the rank condition necessary for the classical Chabauty method fails.

The workshop brought such works to the attention of experts in explicit arithmetic geometry who might be able to contribute ideas towards the project. The mini-course on the non-abelian Chabauty method consisted of the following talks:

- *Explicit aspects of the Chabauty–Kim method* by Jennifer S. Balakrishnan;
- *p-adic heights and integral points on curves* by Amnon Besser;
- *Mixed Tate motives and the $S$-unit equation* by David Corwin;
- *Overview of the Chabauty–Kim method* by Ishai Dan-Cohen; and
- *p-adic heights and rational points on curves* by Netan Dogra.

Two other talks presented alternative $p$-adic approaches to rational points:

- Bas Edixhoven presented a geometric approach to the first non-abelian level of the Chabauty–Kim approach, involving the Poincaré torsor; and
- Brian Lawrence reported on joint work with Akshay Venkatesh using a $p$-adic period map and $p$-adic Hodge theory.

As always in Oberwolfach, the atmosphere was lively and active, providing an ideal environment for the exchange of ideas and productive discussions. The meeting was well-attended, with 53 participants from a variety of backgrounds and seniority. There were 27 talks of various lengths, and ample time was allotted to informal collaboration.

The remaining abstracts included here cover the following areas:

- Arithmetic statistics: Levent Alpoge on the average number of rational points on genus 2 curves, Will Sawin on Cohen–Lenstra heuristics, and Jiuya Wang on Malle’s conjecture.
- Modularity: Frank Calegari on modularity of abelian surfaces, Henri Cohen on modular forms in PARI/GP, John Cremona on using Bianchi newforms to understand elliptic curves of prime conductor over certain quadratic fields, Paul Gunnells on Siegel modular forms, Adam Logan on modular Calabi-Yau 5-folds, Bianca Viray on low-degree points on modular curves, and David Zureick-Brown on rational points on modular curves associated to nonstandard congruence subgroups.
• Characteristic $p$ geometry: Rachel Pries on unlikely intersections between the Torelli locus and Newton polygon strata in the moduli space of principally polarized abelian varieties.

• Integral models of curves: Tim Dokchitser on an almost purely combinatorial approach to understanding regular models of curves, Elisa Lorenzo García on models of nonhyperelliptic curves whose special fiber is hyperelliptic, and Stefan Wewers on a new software project for calculations with models of curves.

• Endomorphisms and isogenies of abelian varieties: E. Victor Flynn on explicit formulas for abelian surfaces with real multiplication by $\sqrt{3}$, John Voight on computing endomorphism rings, and Isabel Vogt on the failure of a local-global principle for the existence of isogenies between elliptic curves.

• Analytic number theory: Michael Bennett on counting primes in an arithmetic progression, and Mark Watkins on two new analytic approaches to the solution to the class number 1 problem.

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