

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Quadratic Forms and Related Structures over Fields

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ABSTRACT. The range of topics discussed at the workshop “Quadratic Forms and Related Structures over Fields” included core themes from the algebraic theory of quadratic and hermitian forms and their Witt groups, several aspects of the theory of linear algebraic groups and homogeneous varieties, cohomological invariants as well as some arithmetic aspects pertaining to the theory of quadratic forms over certain types of ground fields, e.g., function fields.

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Introduction by the Organisers

The workshop was organized by Karim Johannes Becher (Antwerp), Detlev Hoffmann (Dortmund), and Anne Quéguiner-Mathieu (Paris), and was attended by 53 participants. Funding from the Leibniz Association within the grant ‘Oberwolfach Leibniz Graduate Students’ (OWLG) provided support toward the participation of seven young researchers. Additionally, the “US Junior Oberwolfach Fellows” program of the US National Science Foundation funded travel expenses for one post doc from the USA. Finally, Parimala, from Emory University, was supported by the Simons Foundation: she benefited from an extended stay in Europe, spending a month in Paris 13, several days in Lyon, and a week in Dortmund, the first two weeks being partially supported by the ‘Simons Visiting Professors’ program.

The workshop was the thirteenth Oberwolfach meeting on the algebraic theory of quadratic forms and related structures, following a tradition initiated by

Manfred Knebusch, Albrecht Pfister, and Winfried Scharlau in 1975. Throughout the years, the theme of quadratic forms has consistently provided a meeting ground where methods from various areas of mathematics successfully cross-breed. Frequently, results on quadratic and hermitian forms served as test case for far-reaching generalizations. While research emphases have often shifted reflecting current trends, the theory of quadratic forms has absorbed these developments ensuring that its study has stayed timely over the years. Its scope now includes aspects of the theories of algebras with involutions and of linear algebraic groups and their homogeneous spaces over arbitrary fields as well as geometric methods stemming from homotopy and cobordism theories. In addition, the study of quadratic and hermitian forms over specific fields, such as function fields over arithmetic base fields, formally real fields and fields of characteristic 2, has seen quite a resurgence over the last two decades or so and was also the focus of discussions.

The program consisted of 23 talks, including a number of remarkable talks by young participants, who presented impressive results. With the exception of two 30 minute talks, all lectures were scheduled to last 45 minutes. This allowed ample time for questions after each talk. The schedule also included generous recess periods meant to provide more time for less formal research interaction. The participants made full use of this offer by engaging actively in various smaller and often spontaneously formed discussion groups exchanging ideas and knowledge on pertinent workshop related topics.

Whenever possible, an attempt was made to group the talks thematically within a morning or afternoon session. The talks provided an excellent overview of the many exciting developments, new results and current trends in and around the workshop themes and they covered a wide range of topics including, among others, cohomological invariants, local-global principles in various guises, field invariants pertaining to quadratic and hermitian forms, to central simple algebras or to cohomology groups, questions concerning isotropy of quadratic forms or of linear algebraic groups under field extensions, the Grothendieck-Serre conjecture for reductive group schemes over semi-local Dedekind domains, as well as rather novel topics such as alternative Clifford algebras or supertropical quadratic forms.

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