Abstract. The workshop “Geophysical Fluid Dynamics” addressed recent advances in analytical, stochastic, modeling and computational studies of geophysical fluid models. Of central interest were the reduced geophysical models, that are derived by means of asymptotic and scaling techniques, and their investigations by methods from the above disciplines. In particular, contributions concerning the viscous and inviscid geostrophic models, the primitive equations of oceanic and atmospheric dynamics, tropical atmospheric models and their coupling to nonlinear dynamics of phase changes moisture, thermodynamical effects, stratifying effects, as well as boundary layers were presented and discussed.

Mathematics Subject Classification (2010): 76-XX, 86A10, 35-XX, 60Hxx.

Introduction by the Organisers

This workshop was aiming to bringing together experts from diverse scientific disciplines with common interest in geophysical fluid dynamics and to encompass their scientific exchange concerning their investigation of various classes of geophysical fluid models. These models have been of great scientific interest due to the complex structure of their underlying coupled nonlinear dynamics. In particular, numerous scientific tools from mathematical analysis, stochastic dynamics, modeling and computational sciences have been developed to examine their quantitative and qualitative behaviours. As in the case of the Navier-Stokes equations, some of these detailed geophysical models still lack, however, basic understanding concerning global existence and uniqueness of smooth solutions. In oceanic and atmospheric dynamics, as well as in the theory of boundary layers, one often tends...
to derive and investigate reduced simplified models, whose derivations are based on formal asymptotic procedures. These simplified models bring up difficult analytical and physical questions concerning, e.g., the well-posedness, validity and stability of these models for the relevant spatial and temporal scales. Notably, identifying these relevant spatial and temporal scales of validity is already a major mathematical challenge. Thus, a major goal of this workshop was to bridge between recent theoretical advances, in those branches of mathematics relevant to geophysical flows, and the physical understanding of the observed underlying phenomena in those flows; and in particular to judge and validate the reliability of these simplified models for the relevant physical spatial and temporal scales of their derivation.

The mathematical investigation of these models involves many modern mathematical tools ranging from nonlinear partial differential equations and their stochastic counterparts, harmonic analysis, dispersive estimates and transport theory to evolution equations. As a first step one aims to prove the global well-posedness of the underlying equations. This represents also an important step in the development of numerical and computational schemes for simulation of these models.

Of particular interest in this context was also the understanding and analysis of boundary layers, such as Prandtl’s boundary layer model. Furthermore, since certain solutions of the inviscid primitive equations exhibit blow up in finite time, several attempts were made to examine whether the fast rotation term, due to Coriolis force, has a stabilizing effect on these solutions that allows to prolong its life-span and whether certain smoothing techniques, which work well for the three-dimensional Euler equations, may be transferred to this setting. Moreover, modern tropical atmospheric moisture models, taking into account also phase transitions of the clouds’ vapour/water, require the balance laws for energy and entropy. Rigorous verifications of the fact that these models are consistent with the second law of thermodynamics are important. Finally, recently developed downscaling data assimilation algorithms for weather and climate predictions were considered.

The meeting ignited lively and productive interaction and exchange of ideas and was thus a very inspiring experience. Each lecture was allocated 40 minutes followed by a very lively and interactive discussion of 20 minutes. Moreover, the vibrant presence of young participants was very visible during the meeting. In particular, they were encouraged to present their work in a special evening session, which was fully attended by all participants.

In summary, the meeting brought together an excellent balanced mixture of scientists from the various scientific communities. In particular, several leaders from different disciplines met for the first time in person. Notably, the age, gender and geographic diversity of the participants was more than adequate.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”. Moreover, the MFO and the workshop organizers would like to thank the Simons
Foundation for supporting Jinkai Li and Thieu-Huy Nguyen in the “Simons Vis-
it ing Professors” program at the MFO.
### Workshop: Geophysical Fluid Dynamics

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