This topology conference in Oberwolfach was organized by an organizing committee consisting of Mark Behrens, Peter Teichner, Nathalie Wahl and Michael Weiss, the first organizer being new on the team. About 50 mathematicians participated, working in many different areas of algebraic and geometric topology.

The talks were of three types. There were 12 regular one-hour talks, 2 x 2 one-hour talks by the keynote speakers Benson Farb and Dan Isaksen, and 5 half-hour talks.

Farb gave two lectures on representation stability and its applications to arithmetic, while Isaksen talked about motivic homotopy theory and its applications to classical homotopy theory. Both series of lectures where complemented by

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follow-up talks by other speakers. The remaining talks of the conference covered a variety of topics including stable homology of automorphisms of free groups with twisted coefficients and unstable homology of general linear groups, cellular $E_2$–algebras and a motivic version of the $E_2$–operad, 2–Segal spaces, the harmonic compactification of moduli space, the Milnor number, derived induction theory, $K$–theory and $L$–theory for $C^*$–algebras, deformation quantization, higher topological Hochschild homology and topological cyclic homology, Goodwillie calculus for categories and secondary obstructions for positiv scalar curvature. Speakers were instructed to give talks that could be appreciated by an audience of topologists of many different kinds, and they were generally very successful in doing so, also for the shorter talks.

Keynote speaker Benson Farb gave in his first talk a very nice overview of the state of the art of representation stability and the theory of $FI$–modules, a very fruitful theory of Church-Farb-Ellenberg that allows to describe stable phenomena in the homology of sequences of objects that earlier were thought of as having no stability. He then devoted his second lecture on “point counting for topologists”, explaining how the Grothendieck-Lefshetz formula gives a relationship between homological stability for varieties and asymptotic point counting in arithmetic. This idea was the basis of the breakthrough work of Ellenberg-Venkatesh-Westerland about the Cohen-Lenstra heuristics for function fields. These talks were complemented by the talks of Jesse Wolfson and Craig Westerland. Wolfson introduced the concept of “homological densities”, inspired by arithmetic, and described results and conjectures about those from his joint work with Farb and Wood. Westerland gave an account of his joint work with Ellenberg and Tran, where they deduce the asymptotic behaviour of the number of points in certain Hurwitz moduli stack from asymptotic behaviour of the homology of the braid groups with certain twisted coefficients.

Keynote speaker Dan Isaksen started by giving an overview lecture about motivic homotopy theory, explaining how it allows to apply homotopy-theoretic methods to the study of algebraic varieties, and for example better approach their algebraic $K$–theory. This is the circle of ideas that lead to a proof by Voevodsky of the Milnor and Bloch-Kato conjectures which relate the Milnor $K$-theory of a field with étale cohomology. In his second lecture, Isaksen talked about his very recent work where he uses motivic stable homotopy groups to get computations of classical stable homotopy groups of spheres in a much larger range than so far obtained. This was supplemented by the talks of Kirsten Wickelgren and Geoffroy Horel. Wickelgren described her joint work with Kass, where they show that the local degree around an isolated zero in motivic homotopy theory identifies with the degree of a certain quadratic form. She then explained how this allowed to prove new results about the behavior of singularities. Horel, in his talk, constructed a lift of the $E_2$–operad to the category of étale motives over $\mathbb{Q}$.

We now describe the themes of the remaining regular one-hour talks.

Oscar Randal-Williams explained his technique for computing the homology of the automorphisms of free groups with a certain type of twisted coefficients, in
the stable range, a technique that can also be used to compute the homology with twisted coefficients of certain mapping class groups. Alexander Kupers described his joint work with Galatius and Randal-Williams, in which they show that considering families of groups such as general linear groups, automorphisms of free groups or mapping class groups of surfaces as $E_2$–algebras, and studying their $E_2$–homology, gave new information about their ordinary homology. Thomas Nikolaus gave an account of his joint work with Peter Scholze where they give a much simpler construction of topological cyclic homology than what was previously known. This theory was conceived by Bokstedt in the 80’s to approximate algebraic $K$–theory of rings, and was used very successfully for this purpose in particular by Hesselholt-Madsen. On a closely related theme, Birgit Richter gave an overview lecture about topological Hochschild and higher Hochschild homology, explaining its role in studying iterated $K$–theory, but also in distinguishing $A_{\infty}$ structures or detecting ramification of extensions of ring spectra. Nick Rozenblyum gave an operadic framework for studying deformation quantization, and explained how factorization homology allows to quantize mapping spaces with source a manifold by quantizing the target. Gijs Heuts described a “Goodwillie tower” of categories approximating more and more the category of pointed spaces, starting from that of spectra. He showed how this set-up yields a new point of view on the classical equivalence between commutative and Lie algebras in rational homotopy theory, and gives a telescope analogue of this equivalence. Finally, Rudolf Zeidler gave obstructions to families of positive scalar curvature metrics using embedded submanifolds of various codimensions.

The half-hour talks were given in an intense, but very enjoyable, morning session on Wednesday by Daniela Egas, Akhil Mathew, Claudia Scheimbauer, Markus Land and David Sprehn. Egas gave an account of her joint work with Boes on the computation of the homology of the harmonic compactification of the moduli space of Riemann surfaces. Mathew gave a version of Dress induction in equivariant stable homotopy theory (from his joint work with Naumann and Noel). Scheimbauer talked about her joint work with Bergner, Osorno, Ozornova and Rovelli, where they give in particular an equivalence between double categories and 2–Segal sets, with nice examples coming from partial monoids. Land talked about his joint work with Nikolaus, where they study the relationship (or sometimes lack of relationship!) between $K$–theory and $L$–theory. Finally, Sprehn told us about his computation, together with Lahtinen, of non-trivial homology classes in the homology of general linear groups over finite fields at the characteristic. They find non-trivial classes in much lower degrees than previously known.

Once again the Oberwolfach staff, not least the kitchen staff, helped to make this meeting pleasant and memorable. Our thanks go to the institute for creating this atmosphere and making the conference possible.

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Workshop: Topologie

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\textit{Representation Stability: A survey of recent progress} \hspace{1cm} 2018

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