Mathematisches Forschungsinstitut Oberwolfach

Report No. 30/2016

DOI: 10.4171/OWR/2016/30

Hyperbolic Techniques in Modelling, Analysis and Numerics

Organised by
Rinaldo M. Colombo, Brescia
Philippe G. LeFloch, Paris
Christian Rohde, Stuttgart

19 June – 25 June 2016

Abstract. Several research areas are flourishing on the roots of the breakthroughs in conservation laws that took place in the last two decades. The meeting played a key role in providing contacts among the different branches that are currently developing. All the invitees shared the same common background that consists of the analytical and numerical techniques for nonlinear hyperbolic balance laws. However, their fields of applications and their levels of abstraction are very diverse.

The workshop was the unique opportunity to share ideas about analytical issues like the fine-structure of singular solutions or the validity of entropy solution concepts. It turned out that generalized hyperbolic techniques are able to handle the challenges posed by new applications. The design of efficient structure preserving methods turned out to be the major line of development in numerical analysis.

Mathematics Subject Classification (2010): 35L65, 74J40.

Introduction by the Organisers

The workshop *Hyperbolic Techniques in Modelling, Analysis and Numerics*, organized by Rinaldo M. Colombo (Brescia), Phillipe G. LeFloch (Paris) and Christian Rohde (Stuttgart) welcomed 46 invitees from eight different countries. The group of attendants included besides internationally renowned researchers doctoral students and young postdocs. The program consisted of longer comprehensive lectures but also of a small number of short presentations given by young researchers. The general topic of the workshop circled around the mathematical theory of hyperbolic partial differential equations, in particular of balance laws, which has seen an astonishing development in the last two decades. The progress in analysis and

numerics was mainly driven by challenges from continuum mechanics, a prominent role being played by shock waves in gas dynamics. Modeling through hyperbolic partial differential equations has now become a cornerstone in many other branches of science, for instance wherever nonlinear transport phenomena occur. Many new models have been derived, which, in turn, pose completely new questions to the mathematical theory and to the numerical analysis of hyperbolic equations. Given this background, the meeting tried to exploit and foster all possible synergies. Apparently, new joint cooperations can be tracked back to the interaction during the workshop week.

An interesting new development in the field is the characterization of singularity development and transport in different instances of nonlinear wave equations by hyperbolic techniques. Alberto Bressan developped a program to describe the fine structure of sets of generic singularities in a wide class of wave equations. Stefano Bianchini presented new results on a detailed description of the entropy dissipation connected to the emergence of shock waves. Stefano Modena showed how a Lagrangian approach can be used to achieve a deeper insight in the behaviour and the structure of the solutions to hyperbolic conservation laws. Wave breaking in the Hunter-Saxton system was the topic of Anders Nordli. The understanding of the interaction of dispersive approximations and shock waves is far from being a settled problem. Michael Shearer reported on various phenomena related to Korteweg- and Boussinesq-type equations. Sylvie Benzoni demonstrated how modulation theory can help to understand discrete wave motion. Using variational time discretization Michael Westdickenberg gave an existence proof for measure valed solutions of the full Euler system including a characterization of the entropy dissipation. Eitan Tadmor showed how to derive new BV-estimates for the pressureless Euler equations in multiple space dimensions. The talk of Jan Giesselmann on relative entropies for Hamiltonian systems like Euler–Korteweg equations fitted also in this context. Christian Klingenberg broached the issue of the effect of different linearization levels in numerical schemes for multidimensional Euler equations. Even linear wave equations with rough coefficients can pose major difficulties to numerical discretisation methods as was shown by Franziska Weber. This applies even more for hyperbolic systems with uncertainty. Alina Chertock proposed a splitting method for the efficient stochastic Galerkin discretization of Euler systems. The challenges of low Mach number scenarios in astrophysical flows have also been discussed by Christian Klingenberg.

The analytical study of singular limits and associated numerical questions on the design of structure-preserving schemes for asymptotic regimes provided the joint chord for another block of contributions. The study of the zero-viscosity limit, i.e. the passage from a parabolic regularization towards a hyperbolic limit problem, is a seminal topic in the field. Driven by the needs of applied sciences, a much wider variety of asymptotic scenarios is analyzed currently. In this context, Andrea Corli gave a presentation on the study of nonlinear diffusion approximations. Within the workshop, Gianluca Crippa devoted his talk to the passage from non-local to local hyperbolic balance laws. Numerical aspects of non-local evolution equations

have been the topic of Elena Rossi. Graziano Guerra presented rigorous results for the compressible-incompressible passage covering discontinuous solutions. On the numerical side a novel class of asymptotic preserving finite-volume schemes that deal efficiently with weakly compressible low Mach number flows has been advocated by Mária Lukáčová-Medviďová. Manuel Torrilhon reviewed the analysis and numerics for the whole hierarchy of moment systems for the Boltzmann equations. The construction of new stable discretization schemes for moment equation was the topic of the lecture given by Philippe Helluy. Konstantina Trivisa discussed related model hierarchies to describe phenomena of collective self-organization.

As mentioned before, new applications have been always a driving force for the field. Phase transition in compressible two-phase flows has been studied by Ferdinand Thein generalizing the classical Riemann solver concept. Athanasios Tzavaras showed how mixed type models can help to understand shear band instabilities. The control of hyperbolic transport systems for population dynamics has been discussed by Mauro Garavello. The study of in particular hyperbolic evolutions on manifolds and networks is still an emerging research field. This has been addressed in the presentation of Raul Borsche on chemotactic movement on graphs and the lecture of Helge Holden about a hyperbolic transport equation which allows the numerical verification of the well-known Braess paradoxon in traffic flow. Francesca Marcellini showed how hyperbolic Riemann solver techniques can be used to understand the behavior of road traffic at junctions.
Finally the workshop was closed with an overview talk also given by Helge Holden who summarized the state of the art in the field and proposed a number of new challenges.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, "US Junior Oberwolfach Fellows". Moreover, the MFO and the workshop organizers would like to thank the Simons Foundation for supporting Alina Chertock in the "Simons Visiting Professors" program at the MFO.

Workshop: Hyperbolic Techniques in Modelling, Analysis and Numerics

Table of Contents