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Toric Geometry

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Abstract. Toric geometry is a subfield of algebraic geometry with deep intersections with combinatorics. This workshop brought together researchers working in toric geometry, applying toric geometry elsewhere in algebraic geometry, and applying toric geometry elsewhere inside and outside mathematics.

Mathematics Subject Classification (2010): 14M25.

Introduction by the Organisers

Toric geometry is a subfield of algebraic geometry with deep intersections with combinatorics. A toric variety \( X \) is a partial compactification of the algebraic torus \( T \cong (\mathbb{C}^*)^n \) with an action of \( T \) that extends the action of \( T \) on itself. Behind this simple definition, however, is a striking combinatorial dictionary that relates algebro-geometric invariants of the variety \( X \) to geometric-combinatorial invariants of an associated lattice polytope or polyhedral fan. This bridge between the two fields has made toric geometry to an important source of examples and counterexamples in algebraic geometry.

Toric techniques also have applications in other areas, both inside algebraic geometry, in other areas of mathematics, and outside mathematics. Examples inside algebraic geometry include the study of Mori Dream Spaces, varieties with torus actions, Newton-Okounkov bodies, tropical geometry, and degenerations to toric varieties. There are also strong connections to string theory and symplectic geometry, and increasing ties to arithmetic geometry and commutative algebra. Finally, toric varieties also have applications outside mathematics, in areas as diverse as statistics, coding theory, computer modelling, and chemistry.
This workshop brought together researchers working in all aspects of the subject. The talks presented current developments and recent results in “classical” toric geometry, toric-inspired topics, and the use of toric tools in other fields ranging from algebraic geometry via commutative algebra, topology and arithmetic geometry to applications.

Some of the broad themes covered were:

1. Applications to combinatorics (Huh, Katz, Lasoń)
2. Connections to number theory and topology (Gubler, De Cataldo)
3. Applications outside mathematics: Dickenstein (biochemistry), He (physics), Michalek (statistics)
4. Toric inspired algebraic geometry (Brion, Karu, Laface, Satriano)
5. Algebraic aspects (Hering, Kaveh, Smith)
6. Classical toric questions (Altmann, Arzhantsev, Brown, Di Rocco, Grassi, Ilten, Mustaţă, Teissier)

One aspect that we would like to highlight was an evening session on Tuesday of five minute talks by largely junior participants. The session was lively, and began with a five minute talk by Sturmfels, and finished with one by Batyrev. As with most Oberwolfach workshops, the informal conversations that followed the talks, over meals, during coffee breaks, and in the evening, also contributed to a rich scientific week, and we are grateful to Oberwolfach for facilitating that.

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