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Topological Recursion and TQFTs

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Abstract. The topological recursion is an ubiquitous structure in enumerative geometry of surfaces and topological quantum field theories. Since its invention in the context of matrix models, it has been found or conjectured to compute intersection numbers in the moduli space of curves, topological string amplitudes, asymptotics of knot invariants, and more generally semiclassical expansion in topological quantum field theories. This workshop brought together mathematicians and theoretical physicists with various background to understand better the underlying geometry, learn about recent advances (notably on quantisation of spectral curves, topological strings and quantum gauge theories, and geometry of moduli spaces) and discuss the hot topics in the area.

Mathematics Subject Classification (2010): 4Nxx, 14H60, 14H70, 37K10, 51PXX, 53Zxx, 81Qxx, 81Txx.

Introduction by the Organisers

At the core of the topological recursion procedure lies the notion of spectral curve: It is a Lagrangian curve $\mathcal{C}$ in $\mathbb{C} \times \mathbb{C}$, a one-form $\omega_{0,1}$ on $\mathcal{C}$ which is the restriction of an antiderivative of the symplectic form on $\mathbb{C} \times \mathbb{C}$, and a bi-form $\omega_{0,2}$ on $\mathcal{C}^2$ allowing a form-cycle duality on $\mathcal{C}$. Out of $\omega_{0,1}$ and $\omega_{0,2}$, it defines a sequence of n-forms $\omega_{g,n}$ on $\mathcal{C}^n$ (the correlators) by a recursion on $2g-2+n > 0$, and a sequence of numbers $\omega_{g,0}$ (the free energies). This definition is made to solve a set of loop equations, which are closely related to the Virasoro constraints. Already for simple examples of spectral curves, the $\omega_{g,n}$'s encode interesting geometric information, e.g. the intersection number of ψ classes on the moduli space $\mathcal{M}_{g,n}$ of genus g Riemann surfaces with n punctures, or numbers of coverings of the sphere by genus g surfaces (simple Hurwitz numbers). In general, the $(\omega_{g,n})_{g,n}$ have many interesting properties: Seiberg-Witten like relations for the variations of initial data, symplectic invariance (change of antiderivative $\omega_{0,1}$), modular properties/holomorphic anomaly in relation with deformations of $\omega_{0,2}$, explicit representation of $\omega_{g,n}$ in terms of integrals of tautological classes on $\overline{\mathcal{M}}_{g,n}$, etc.

This definition is strikingly universal, and has found a broad range of applications in the last 10 years, that motivated our workshop. It appears for instance in large size asymptotic expansion in Hermitean matrix models, $\hbar \to 0$ asymptotic expansion in integrable systems, and in enumerative geometry of surfaces. The latter is ubiquitous in mathematical physics, and includes random maps ($2d$ quantum gravity), invariants of 3-manifolds (Kontsevich integral, Chern-Simons theory, for example), topological string theory, gauge theories, etc. This commonality comes from the (sometimes unprecise) observation that, behind all those problems, there exist Feynman diagrams embedded on surfaces. The general properties of the topological recursion often have interesting interpretations in the problem where it is applied: construction of partition function which are automorphic forms, symplectic invariance seen as framing independence of the closed string sector, ELSV type formulae for enumerative problems, and many more.

The developments of the theory of the topological recursion and its relations to many problems of topological quantum field theory have formed the main topic of the workshop. It aimed at a better abstract understanding of the underlying geometry.

This motivated the presence of many specialists of quantum field theories from mathematics and theoretical physics who are not always working with topological recursion, but are handling problems of related interest: topological strings (Alim, Kashaev, Klemm), gauge theories (Dimofte, Hollands, Scheidegger, Teschner), deformation quantization (Petit), geometry of moduli spaces (Andersen, Mulase). Two other important round of topics which exhibited recent advances were especially discussed: quantum curves (Bouchard, Belliard, Petit, Sulkowski, Mulase) and Frobenius manifolds & cohomological field theories (Dunin-Barkowski, Do, Orantin, Milanov), with applications to enumerative geometry and integrable systems. The workshop was an occasion to diffuse those ideas to a broader community.

The workshop counted 27 participants (including the organizers), from all over Europe, Canada, the US, Australia and Japan. They were a balanced mix of established senior scientists and younger researchers, as well as 3 postdocs and Ph.D. students (Belliard, Dunin-Barkowski, Zenkevich). In order to boost scientific exchange, a problem session was arranged to probe still not completely shaped ideas. Moreover, a special, rather informal, evening session joint with the parallel workshop on *Hochschild Cohomology in Algebra, Geometry, and Topology* was organised with three short talks from each side. It was a very good idea to have this other mini-workshop at the same time as ours as many scientific discussions (e.g. on Hochschild cohomology and higher categories) relevant for the topic of our workshop, have resulted from it.

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Workshop: Topological Recursion and TQFTs

Table of Contents