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Topological Recursion and TQFTs

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ABSTRACT. The topological recursion is an ubiquitous structure in enumerative geometry of surfaces and topological quantum field theories. Since its invention in the context of matrix models, it has been found or conjectured to compute intersection numbers in the moduli space of curves, topological string amplitudes, asymptotics of knot invariants, and more generally semiclassical expansion in topological quantum field theories. This workshop brought together mathematicians and theoretical physicists with various background to understand better the underlying geometry, learn about recent advances (notably on quantisation of spectral curves, topological strings and quantum gauge theories, and geometry of moduli spaces) and discuss the hot topics in the area.

Mathematics Subject Classification (2010): 4Nxx, 14H60, 14H70, 37K10, 51PXX, 53Zxx, 81Qxx, 81Txx.

Introduction by the Organisers

At the core of the topological recursion procedure lies the notion of spectral curve: It is a Lagrangian curve \mathcal{C} in $\mathbb{C} \times \mathbb{C}$, a one-form $\omega_{0,1}$ on \mathcal{C} which is the restriction of an antiderivative of the symplectic form on $\mathbb{C} \times \mathbb{C}$, and a bi-form $\omega_{0,2}$ on \mathcal{C}^2 allowing a form-cycle duality on \mathcal{C} . Out of $\omega_{0,1}$ and $\omega_{0,2}$, it defines a sequence of n -forms $\omega_{g,n}$ on \mathcal{C}^n (the correlators) by a recursion on $2g - 2 + n > 0$, and a sequence of numbers $\omega_{g,0}$ (the free energies). This definition is made to solve a set of loop equations, which are closely related to the Virasoro constraints. Already for simple examples of spectral curves, the $\omega_{g,n}$'s encode interesting geometric information,

e.g. the intersection number of ψ classes on the moduli space $\mathcal{M}_{g,n}$ of genus g Riemann surfaces with n punctures, or numbers of coverings of the sphere by genus g surfaces (simple Hurwitz numbers). In general, the $(\omega_{g,n})_{g,n}$ have many interesting properties: Seiberg-Witten like relations for the variations of initial data, symplectic invariance (change of antiderivative $\omega_{0,1}$), modular properties/holomorphic anomaly in relation with deformations of $\omega_{0,2}$, explicit representation of $\omega_{g,n}$ in terms of integrals of tautological classes on $\overline{\mathcal{M}}_{g,n}$, etc.

This definition is strikingly universal, and has found a broad range of applications in the last 10 years, that motivated our workshop. It appears for instance in large size asymptotic expansion in Hermitean matrix models, $\hbar \rightarrow 0$ asymptotic expansion in integrable systems, and in enumerative geometry of surfaces. The latter is ubiquitous in mathematical physics, and includes random maps ($2d$ quantum gravity), invariants of 3-manifolds (Kontsevich integral, Chern-Simons theory, for example), topological string theory, gauge theories, etc. This commonality comes from the (sometimes unprecise) observation that, behind all those problems, there exist Feynman diagrams embedded on surfaces. The general properties of the topological recursion often have interesting interpretations in the problem where it is applied: construction of partition function which are automorphic forms, symplectic invariance seen as framing independence of the closed string sector, ELSV type formulae for enumerative problems, and many more.

The developments of the theory of the topological recursion and its relations to many problems of topological quantum field theory have formed the main topic of the workshop. It aimed at a better abstract understanding of the underlying geometry.

This motivated the presence of many specialists of quantum field theories from mathematics and theoretical physics who are not always working with topological recursion, but are handling problems of related interest: topological strings (Alim, Kashaev, Klemm), gauge theories (Dimofte, Hollands, Scheidegger, Teschner), deformation quantization (Petit), geometry of moduli spaces (Andersen, Mulase). Two other important round of topics which exhibited recent advances were especially discussed: quantum curves (Bouchard, Belliard, Petit, Sulkowski, Mulase) and Frobenius manifolds & cohomological field theories (Dunin-Barkowski, Do, Orantin, Milanov), with applications to enumerative geometry and integrable systems. The workshop was an occasion to diffuse those ideas to a broader community.

The workshop counted 27 participants (including the organizers), from all over Europe, Canada, the US, Australia and Japan. They were a balanced mix of established senior scientists and younger researchers, as well as 3 postdocs and Ph.D. students (Belliard, Dunin-Barkowski, Zenkevich). In order to boost scientific exchange, a problem session was arranged to probe still not completely shaped ideas. Moreover, a special, rather informal, evening session joint with the parallel workshop on *Hochschild Cohomology in Algebra, Geometry, and Topology* was organised with three short talks from each side. It was a very good idea to have this other mini-workshop at the same time as ours as many scientific discussions

(e.g. on Hochschild cohomology and higher categories) relevant for the topic of our workshop, have resulted from it.

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Workshop: Topological Recursion and TQFTs**Table of Contents**

Murad Alim (joint with S.T. Yau and J. Zhou)	
<i>Geometric hints of non-perturbative topological strings</i>	393
Jørgen Ellegaard Andersen	
<i>Geometric quantisation of moduli space, conformal field theory and modular functors</i>	396
Raphaël Belliard (joint with Bertrand Eynard, Olivier Marchal)	
<i>Loop equations from differential systems</i>	399
Vincent Bouchard (joint with Bertrand Eynard)	
<i>Quantum curves and topological recursion</i>	400
Tudor Dimofte (joint with Stavros Garoufalidis, Don Zagier)	
<i>Quantum modularity and Chern-Simons theory</i>	401
Norman Do (joint with Paul Norbury)	
<i>Topological recursion for irregular spectral curves (A tale of two tau-functions)</i>	403
Olivia Dumitrescu (joint with Motohico Mulase)	
<i>From cellular graphs to Topological Quantum Field Theories</i>	406
Petr Dunin-Barkowski (joint with Paul Norbury, Nicolas Orantin, Alexandr Popolitov, Sergey Shadrin)	
<i>Dubrovin's superpotential as a global spectral curve</i>	410
Lotte Hollands (joint with Andy Neitzke)	
<i>Twisted superpotential of the T_3 theory</i>	413
Jacques Hurtubise (joint with Christiane Rousseau)	
<i>Deforming irregular singular points</i>	415
Rinat Kashaev (joint with Marcos Mariño, Szabolcs Zakany)	
<i>The quantum dilogarithm and operators from topological strings</i>	417
Albrecht Klemm	
<i>Elliptically fibered Calabi-Yau manifolds and the ring of weak Jacobi forms</i>	420
Todor Milanov	
<i>Vertex algebras and the topological recursion for A_N singularity</i>	425
Motohico Mulase (joint with Olivia Dumitrescu, Laura Fredrickson, Georgios Kydonakis, Rafe Mazzeo, Andrew Neitzke)	
<i>On Gaiotto conjecture</i>	428

Nicolas Orantin	
<i>Topological recursion and cohomological field theories</i>	429
François Petit	
<i>Quantization of spectral curves and DQ-modules</i>	432
Jörg Teschner	
<i>Quantisations of some spectral curves</i>	433
Emanuel Scheidegger (joint with Johanna Knapp, Mauricio Romo)	
<i>Hemisphere Partition Function and Gauged Linear Sigma Model</i>	436
Michael Shapiro (joint with Anna Felikson, Pavel Tumarkin)	
<i>Quivers of finite mutation type</i>	440
Piotr Sułkowski (joint with Masahide Manabe)	
<i>Quantum curves, matrix models and conformal field theory</i>	440
Yegor Zenkevich (joint with Alexei Morozov)	
<i>Refined topological strings and conformal blocks</i>	443