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Arbeitsgemeinschaft: Mathematical Quasicrystals

Organised by
Alan Haynes, York
Rodrigo Treviño, New York
Barak Weiss, Tel Aviv

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Abstract. This introductory workshop encouraged participants to read important recent works in the topology, geometry and dynamics of highly regular (but aperiodic) discrete sets in Euclidean spaces, and their corresponding tiling spaces. These sets have been recently under intensive investigation by researchers in topology, mathematical physics, dynamics, diophantine approximation, and discrete mathematics, and various different perspectives were emphasized.

Mathematics Subject Classification (2010): 52C23.

Introduction by the Organisers

The mathematical study of aperiodic tilings and aperiodic discrete sets in $\mathbb{R}^d$ began with the discovery in the 1960s by Wang and Berger, followed by Robinson and Penrose in the 1970s, of a finite set of tiles which tile $\mathbb{R}^2$ aperiodically. These studies later received physical motivation with the discovery in 1982 by Schechtman et al. of materials which do not have crystalline structures. These structures were called quasicrystals by Levine and Steinhardt.

In recent years much intensive work has been devoted to investigating the large scale geometry of discrete subsets of $\mathbb{R}^d$. The sets which are studied are typically not periodic, but share some of the properties of periodic sets, corresponding to various weakenings of the notion of periodicity. As a consequence they have come to be studied under the loosely defined term quasicrystals. The study of such sets has a long history in different mathematical disciplines, such as dynamics (in connection with cross-sections for continuous group actions, virtual subgroups), mathematical physics (quasicrystals and almost periodic structures,

questions of diffraction), operator algebras and K-theory (cohomology theories for pattern spaces, solenoids and laminated spaces), geometric group theory (quasi-isometries and coarse isometries, almost subgroups), and geometric combinatorics (packing and covering questions). Although researchers from different disciplines are interested in different questions, there are many connections between work being done by different groups of people. The goal of this meeting is to provide an introduction to some of the main examples and questions surrounding mathematical quasicrystals, making it possible to bridge the cultural gaps between people studying the same objects from different points of view.

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