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Geometric Topology

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Abstract. Geometric topology has seen significant advances in the understanding and application of infinite symmetries and of the principles behind them. On the one hand, for advances in (geometric) group theory, tools from algebraic topology are applied and extended; on the other hand, spectacular results in topology (e.g., the proofs of new cases of the Novikov conjecture or the Atiyah conjecture) were only possible through a combination of methods of homotopy theory and new insights in the geometry of groups. This workshop focused on the rich interplay between algebraic topology and geometric group theory.

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Introduction by the Organisers

Geometric topology has seen significant advances in the understanding and application of infinite symmetries and of the principles behind them. This workshop focused on the rich interplay between algebraic topology and geometric group theory. The research fields of the 53 participants of the workshop covered homotopy theory, manifold topology, low-dimensional topology, geometric group theory, and geometry of topological groups.

Some of the main topics of the workshop were:

- Variations of hyperbolicity
- Rigidity versus flexibility of geometry and topology
- Homological properties of manifolds
- Bounded cohomology and its applications
- Classification of groups and their representations by geometric means

This wide range of interests is united on several levels: The type of problems considered and aspired goals of research are driven by related ideas (e.g., rigidity phenomena). Also the tools and techniques are shared (e.g., the language of algebraic topology, probabilistic methods). Moreover, the topics listed above are interrelated in various ways (e.g., classically, hyperbolicity is a strong indicator for rigidity). The workshop offered the opportunity to strengthen the bonds between these fields.

The formal part of the programme consisted of twenty regular research talks and a *Gong Show* of 10 minutes talks by eleven PhD students and recent postdocs. This formal part was complemented by a variety of lively discussions in smaller groups.

On the one hand, the research talks communicated and documented the current state of the art. On the other hand, many of the talks also advertised open problems linking topology and group theory. For example, Ian Leary proposed further variations of the question by Eilenberg and Ganea on the relation between cohomological dimension and geometric dimension of groups; and Kevin Schreve proposed the *action dimension conjecture* bounding the minimal dimension of a contractible manifold on which a group acts by twice the L^2-cohomological dimension. This is a relative of the Singer conjecture, which predicts concentration of L^2-cohomology in middle degree of an aspherical manifold. Michał Marcinkowski advertised new candidates (constructed via Davis' asphericalization construction and surgery) for counterexamples of Gromov's macroscopic dimension conjecture (which bounds the macroscopic dimension of an n-dimensional manifold with positive scalar curvature by $n-2$).

We will now describe the main topics and some selected recent achievements that were discussed during this workshop in more detail:

Extended notions of hyperbolicity. Because of its strong relation with rigidity, many attempts have been made in the past to vary the notion of hyperbolicity. A particularly versatile generalisation of hyperbolicity for groups is acylindrical hyperbolicity, introduced by Denis Osin. Recently, Denis Osin (in joint work with Hull) showed that acylindrically hyperbolic groups with trivial finite radical are highly transitive, and hence do not satisfy mixed identities (a notion related to universal equivalence of groups). Another type of variation of hyperbolicity was proposed by Bogdan Nica (in joint work with Jan Spakula): They introduced superbolicity as a concept conveniently interpolating between CAT(-1)-spaces and Green metrics on hyperbolic groups on the one hand and good hyperbolicity and strong bolicity on the other hand.

Rigidity versus flexibility of geometry and topology. Rigidity phenomena in geometry, topology, and group theory have significantly shaped and fused these fields. A prominent example is the Baum-Connes conjecture connecting these fields via operator algebras. Using a graphical small cancellation technique and probabilistic tools, Damian Osajda constructed finitely generated groups that contain *isometrically* embedded (into specific Cayley graphs) expanders. Topologically, these groups lead to examples of aspherical manifolds whose fundamental groups contain quasi-isometrically embedded expanders. Groups of this type provide an interesting source of counterexamples, e.g., in the context of the Baum-Connes conjecture.

Romain Tessera studied a new geometric relation on infinite groups that lies between the (purely algebraic) commensurability and the (very flexible) quasi-isometry and showed its success on generalized Baumslag-Solitar groups.

L^2-Invariants and their applications. L^2-invariants, in particular L^2-Betti numbers, L^2-torsion, and Novikov-Shubin invariants are a powerful and well-established toolbox of invariants. Many aspects of them, on the other hand, still remain mysterious.

Łukasz Grabowski presented the first counterexamples to a conjecture of Lott and Lück; showing that there are manifolds such that some of the Novikov-Shubin invariants are equal to 0. This is particularly important in light of the fact that their positivity used to be a standard assumption in the treatment of secondary L^2-invariants, in particular the L^2-torsion.

On the other hand, by now it is known that this condition can be replaced by the much weaker condition of "L^2-determinant class", which is known to be satisfied in many cases. The development and use of L^2-torsion therefore remains meaningful and is vigorously carried out.

Wolfgang Lück reported on a new development here: the twisted L^2-torsion function, which is a powerful invariant. He, and also Stefan Friedl, presented a host of structural and computational results of this new invariant, in particular for 3-manifolds. It turns out to recover the hyperbolic volume, also the Thurston norm, and the information contained in the classical Alexander polynomial and modern twisted version of it. Stefan Friedl, on the other hand, explained an explicit and easy combinatorial algorithm to compute the Thurston norm for certain classes of 3-manifolds; This algorithm is based on associated polyhedra that also play a role in the calculation of the L^2-torsion function.

Bounded cohomology and its applications. Bounded cohomology in higher degrees has remained rather mysterious in the past decades. However, in recent years, a young community has evolved and taken first, promising steps to approach bounded cohomology in higher degrees. For example, Tobias Hartnick and Andreas Ott related problems in bounded cohomology to partial differential equations and proved that continuous bounded cohomology of $\mathrm{SL}(2,\mathbb{R})$ in degree four is trivial, thereby confirming a conjecture of Monod in a special case. Michelle Bucher-Karlsson (in joint work with Burger and Iozzi) adapted an explicit construction by Goncharov to prove that the comparison map between continuous bounded cohomology and continuous cohomology of $\mathrm{PSL}(n,\mathbb{C})$ is an isomorphism in degree 3, thereby proving a classical conjecture by Dupont in a special case. This result is not only interesting in its own right, but also has applications to rigidity of volume representations of hyperbolic 3-manifolds. Both of the above approaches to continuous bounded cohomology have the potential to generalise to further higher degrees. For discrete groups, Roberto Frigerio (in joint work with Pozzetti and Sisto) extended the framework of quasi-morphisms to higher degrees and combined this with hyperbolically embedded subgroups, which gives interesting inheritance results for bounded cohomology in higher degrees.

All of these topics are also related to the *classification of groups and their representations by geometric means*.

The Mathematische Forschungsinstitut Oberwolfach provided an excellent environment and inspiring atmosphere for this workshop and we are grateful for its hospitality.

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