Abstract. The aim of the meeting is to discuss several classes of Schrödinger equations appearing within the Ginzburg-Landau theory of superconductivity. The related problems are discussed from several perspectives including semiclassical analysis, PDE in non-smooth domains, geometric spectral theory and operator theory, which should provide a new insight into various phenomena appearing in superconducting systems.


Introduction by the Organisers

The idea of the meeting was to discuss in a concentrated way several particular classes of differential operators appearing in the theory of surface superconductivity. We mention explicitly the two most important representatives, which served as an initial motivation. Let Ω be a bounded domain in \(\mathbb{R}^n\) and \(n\) be the outward pointing unit normal vector at its boundary \(\partial \Omega\) which is assumed to be sufficiently regular. If \(A\) is a magnetic vector potential and \(\lambda\) is a coupling constant, one is interested in the associated magnetic Neumann eigenvalue problem

\[
(i\nabla + \lambda A)^2 u = Eu \text{ in } \Omega, \quad n \cdot (i\nabla + \lambda A)u = 0 \text{ at } \partial \Omega.
\]

Another important example is the (zero-field) Robin eigenvalue problem

\[
-\Delta u = Eu \text{ in } \Omega, \quad n \cdot \nabla u = \lambda u \text{ at } \partial \Omega.
\]

The both problems may be obtained through the linearization of the respective Ginzburg-Landau functionals, and the lowest eigenvalues \(E = E(\lambda)\) are known
be related to the critical temperature at which the normal state becomes unstable, while the respective eigenfunction $u$ describes the associated density of the particles (Cooper pairs). In view of this correspondence, one is interested in the dependence of the eigenvalue and the eigenfunction on the geometry of $\Omega$ and the coupling constant $\lambda$. A considerable amount of literature is dedicated to the study of the problem (1) in the limit $\lambda \to +\infty$, which is based on various advanced tools from the semiclassical and pseudodifferential analysis. It appears that the respective eigenfunctions concentrate near the boundary, whose geometric properties determine the eigenvalue asymptotics. During the last years, a particular attention is given to domains whose boundaries have singularities like corners or edges, and in that case the study of new classes of model domains, such as sectors or cones, becomes of importance.

During recent contacts at various meetings we found out that very similar qualitative effects are valid for the Robin problem (2) in the strong coupling limit. Actually, the study of the Robin eigenvalue asymptotics appeared first in the study of reaction-diffusion equations, and its relevance to the theory of superconductivity was given a limited attention only. On the other hand, recently, it has attracted an increasing attention from the point of view of the shape optimization, numerical computation or non-smoothness effects. One may also mention the recent applications to the classical topics such as the extension theorems for Sobolev spaces or Hardy inequalities.

In view of the above observations, we invited a group of experts from different scientific communities in order to exchange new ideas and methods concerning the analysis of differential operators of the above types. The meeting was concentrated on several specific questions: such as the qualitative spectral theory of linear differential operators in bounded and unbounded domains, optimization of eigenvalues with respect to geometry, boundary value problems in non-smooth domains, semiclassical methods. Some non-linear models were discussed as well.

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Mini-Workshop: Eigenvalue Problems in Surface Superconductivity

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